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Abstract

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POLARIZATION EFFECTS IN PARTICLE PRODUCTION IN COLLIDING-BEAM EXPERIMENTS

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1. As has become clear recently ([1]), electrons moving in a homogeneous magnetic field become polarized as a result of radiation. The resulting polarization is transverse, so that the spin of the electrons will be directed opposite to the direction of the magnetic field, and the spin of the positron along the direction of the magnetic field. As for longitudinal polarization, it does not arise, in particular because, owing to the presence of the electron' s anomalous magnetic moment, the spin precesses in the external magnetic field. The characteristic polarization time turns out to be equal to

$$\tau^{-1} = \frac{5\sqrt{3}}{8} me^2 \left(\frac{E}{m}\right)^2 \left(\frac{H}{H_0}\right)^3, \tag{1}$$

where $H_0 = m^2/e = 4.7 \cdot 10^{13}$ oersted.

If one takes $H = 1.5 \cdot 10^4$ oersted, $E = 1$ BeV, then $\tau = 18$ min; this time is of the same order as the time during which storage rings with accumulated electrons will operate. Although the question of the process of electron polarization under the action of radiation in real storage rings must certainly still be resolved, it is quite probable that electrons in storage rings will be at least partially polarized. At the same time, estimates of depolarization effects show that they are small at energies below several BeV.

It is therefore of interest to estimate the influence of electron polarization on the principal two-particle processes occurring in the annihilation of an electron-positron pair. We note that summation over the spins of the final particles is carried out, since the study of the polarizations of the final particles is a much more complicated problem.

Below are given the cross sections for the production of a pair of pions, muons, nucleons, and vector mesons by polarized electrons and positrons. The cross section for elastic scattering of polarized electron and positron is contained in ([2]), and the cross section for two-photon annihilation of polarized electron and positron is given in ([3]).

2. The cross section for two-particle annihilation of polarized electron and positron in the one-photon approximation may be represented in the form

$$d\sigma_2 = \frac{\alpha^2}{P^4}\,\frac{1}{\sqrt{(pp_1)^2 - m^4}}\,X\,\frac{|\mathbf{q}_1|}{E_q}\left|\frac{dE_f}{dE_{q_1}}\right|^{-1} d\Omega_{\mathbf{q}1}. \tag{2}$$

We have introduced the following notation: p, p_1 are the momenta of the initial electron and positron; s, s_1 are their spin vectors; $P = p + p_1$; $K = q - q_1$, where q and q_1 are the momenta of the final particles; m is the electron mass; M is the mass of the produced particles.

In the case of two-pion annihilation we have:

$$\begin{aligned} X_{2\pi} = \frac{|F_{2\pi}(P^2)|^2}{4}&\{[2(Kp)(Kp_1) - K^2(m^2 + pp_1)][1 - (ss_1)] \\ &- 2(Ks)(Ks_1)[m^2 + (pp_1)] + 2(p_1 s)(Ks_1)(pK) \\ &+ 2(ps_1)(Ks)(p_1 K) - K^2(p_1 s)(ps_1)\}; \end{aligned} \tag{3}$$

where $F_{2\pi}(P^2)$ is the pion form factor, complex for $P^2 > 4M^2$. In the center-of-mass system the cross section can be written in the form

$$\begin{aligned} \sigma_{2\pi}(\vartheta) = \frac{\alpha^2|F_{2\pi}(4E^2)|^2}{32E^2}\,\frac{|\mathbf{q}|}{|\mathbf{p}|E^2}\Bigg\{\mathbf{q}^2\left(1 - \frac{\mathbf{p}^2}{E^2}\cos^2\vartheta\right)\left[1 + (\vec{\zeta}\vec{\zeta}_1)\right] \\ - 2(\mathbf{q}\vec{\zeta})(\mathbf{q}\vec{\zeta}_1) + \frac{2(\mathbf{pq})}{E(E+m)}\left[(\mathbf{q}\vec{\zeta}_1)(\mathbf{p}\vec{\zeta}) + (\mathbf{q}\vec{\zeta})(\mathbf{p}\vec{\zeta}_1)\right] - \frac{2(\mathbf{pq})^2}{E^2(E+m)^2}(\vec{\zeta}\mathbf{p})(\vec{\zeta}_1\mathbf{p})\Bigg\}, \end{aligned} \tag{4}$$

Here E is the energy of the particles; $\mathbf{p}$ is the momentum of the initial electron; $\mathbf{q}$ is the momentum of the final pion; ϑ is the angle between the directions of motion of the electron and the pion; $\vec{\zeta}, \vec{\zeta}_1$ are the polarizations of the electron and positron in their rest frame.

Let us consider the practically important case of transverse antiparallel polarization of the initial particles, when $(\vec{\zeta}\mathbf{p}) = -(\vec{\zeta}_1\mathbf{p}) = 0$. In this case we obtain

$$d\sigma_{2\pi} = d\sigma^0_{2\pi}\left[1 + |\vec{\zeta}|\,|\vec{\zeta}_1|\,(2\sin^2\varphi - 1)\right] d\Omega, \tag{5}$$

where $d\sigma^0_{2\pi}$ is the cross section of the process for unpolarized initial particles; φ is the angle between the production plane (the plane passing through the vectors $\mathbf{p}, \mathbf{q}$) and the plane of the orbit. In the case of 100% polarization $|\vec{\zeta}| = |\vec{\zeta}_1| = 1$; then it is seen that the cross section $d\sigma_{2\pi} = 0$ if $\varphi = 0$ (the production plane coincides with the plane of the orbit), and $d\sigma_{2\pi} = 2d\sigma^0_{2\pi}$ if $\varphi = \pi/2$ (the production plane is perpendicular to the plane of the orbit).

3. Let us consider the cross section for the production of a fermion pair. In this case

$$X_{2f} = 2D_1 \left[m^2 \left(1 - (ss_1)\right) + \frac{P^2}{2} \right] +$$
$$+2D_2 \left\{ \left[(qp_1)^2 + (qp)^2 + \frac{P^2}{2} \left(M^2 - \frac{P^2}{2} \right) \right] \left(1 - (ss_1)\right) \right.$$
$$+ M^2 \left[(sq)(s_1 q) + (sq_1)(s_1 q_1) \right] + (s_1 q_1)(sq) \left[M^2 - P^2 + 2(pq) \right]$$
$$\left. + (s_1 q)(sq_1) \left[M^2 - P^2 + 2(qp_1) \right] \right\}, \qquad (6)$$

where

$$D_1 = \frac{P^2}{2} |F_1 + \mu F_2|^2; \qquad D_2 = |F_1|^2 - \frac{P^2 \mu^2}{4M^2} |F_2|^2;$$

μ is the anomalous magnetic moment of the fermion; F_1 and F_2 are the electric and magnetic form factors of the fermion, defined in the standard way.

In the center-of-mass system we have:

$$\sigma_{2f}(\vartheta) = \frac{\alpha^2}{32E^2} \frac{|\mathbf{q}|}{|\mathbf{p}|E^2} \left\{ D_1 \left[2 + \frac{m^2}{E^2} \left(1 + (\vec{\zeta}\vec{\zeta}_1)\right) + \frac{2}{E^2} (\vec{\zeta}\mathbf{p})(\vec{\zeta}_1 \mathbf{p}) \right] \right.$$
$$- 2D_2 \left[\mathbf{q}^2 \left(1 - \frac{\mathbf{p}^2}{E^2} \cos^2 \vartheta \right) \left(1 + (\vec{\zeta}_1 \vec{\zeta})\right) - 2(\mathbf{q}\vec{\zeta})(\mathbf{q}\vec{\zeta}_1) \right.$$
$$\left. \left. + \frac{2(\mathbf{pq})}{E(E+m)} \left[(\mathbf{q}\vec{\zeta}_1)(\mathbf{p}\vec{\zeta}) + (\mathbf{q}\vec{\zeta})(\mathbf{p}\vec{\zeta}_1) \right] - \frac{2(\mathbf{pq})^2}{E^2(E+m)^2} (\vec{\zeta}\mathbf{p})(\vec{\zeta}_1 \mathbf{p}) \right] \right\}, \qquad (7)$$

where the notation is the same as in formula (4).

This cross section consists of two parts: a "scalar" part, equal to the cross section for two-pion annihilation multiplied by $(-2D_2)$, and an "isotropic" part, independent of the direction of the momenta of the produced particles. Such a decomposition, as will be shown below, is a consequence of the law of conservation of current, of relativistic and charge invariance, and holds for all particles, independently of their spin.

The cross sections given describe the process of production of a muon pair and of a baryon pair. In the case of muon production the form factor F_2 may be set equal to 0; in the case of production of a nucleon pair, F_1 and F_2 are continuations into the timelike region of momentum transfer of the electric

and magnetic form factors of the nucleon, determined in experiments on the scattering of electrons by nucleons. These form factors in our case are complex.

It is possible that there exist energies at which $D_2 = 0$ ($|F_1|^2 = (E^2\mu^2/M^2)|F_2|^2$); at this point the cross section becomes completely isotropic. In the actual case of transverse antiparallel polarization we have

$$\sigma_{2f}(\vartheta) = \frac{\alpha^2}{16E^2}\frac{|\mathbf{q}|}{E}\left\{2|F_1+\mu F_2|^2 - \frac{\mathbf{q}^2}{E^2}\sin^2\vartheta\left[|F_1|^2 - \frac{E^2\mu^2}{M^2}|F_2|^2\right]\times \right.$$
$$\left. \times\left[1+|\vec{\xi}|\,|\vec{\xi}_1|(2\sin^2\varphi-1)\right]\right\}. \tag{8}$$

As is seen, the "scalar" part is strongly deformed depending on the angle φ, whereas the "isotropic" part does not depend on it at all. As a result, the deformation of the cross section as a whole when the angle φ is varied will be weaker than in the case of two-pion annihilation.

Let us now turn to annihilation into a pair of vector particles. The most general form of the transition current satisfying the requirements of relativistic and charge invariance, as well as the law of current conservation, is

$$M_\mu(q,t;q_1,t_1) = -\frac{e}{2\sqrt{q_0 q_{10}}}\left\{K_\mu\left[(tt_1)f(P^2) + (Kt)(Kt_1)g(P^2)\right] + \right.$$
$$\left. + \left[t_\mu(Pt_1) - t_{1\mu}(Pt)\right]h(P^2)\right\}. \tag{9}$$

Here t, t_1 are the polarizations of the vector particle; $f(P^2)$, $g(P^2)$, $h(P^2)$ are form factors. To sum over the polarizations of the vector particles one may use the following device. From the same general requirements for which formula (9) was written, it follows that the sum over polarizations is

$$\sum_{t_1 t_2} M_\mu M_\nu^* = \frac{1}{4q_0 q_{10}}\left[4D_1(P^2)\left(\frac{P_\mu P_\nu}{P^2} - g_{\mu\nu}\right) - 2D_2(P^2)K_\mu K_\nu\right], \tag{10}$$

where the functions $D_1(P^2)$, $D_2(P^2)$ can be determined by contracting the tensor obtained, for example, with the tensors $g^{\mu\nu}$ and $K^\mu K^\nu$, contracting the left-hand side correspondingly and solving the equations obtained for D_1 and D_2.

From the very derivation of the right-hand side of formula (10) it is clear that its tensor form does not depend on the spin of the particle (although the functions D_1 and D_2 themselves are, of course, determined by the spin). Since this tensor is contracted with the universal tensor of the electron–positron part for the given problem, the form of the cross section proves to be universal and is given by formulas (6) and (7).

The form factors f, g, h introduced in formula (9) are related to the electric, and to the linear combination of the magnetic and quadrupole, form factors of the vector particle as follows:

$$f = -\left[G_1 + \frac{P^2}{2M^2}\varepsilon_B G_3\right], \qquad g = -\frac{\varepsilon_B G_3}{M^2}, \qquad h = \left[G_1 + \mu_B G_2 + \varepsilon_B G_3\right], \tag{11}$$

where the static anomalous magnetic moment of the vector particle is $\mu_B + \varepsilon_B$, and the static anomalous electric quadrupole moment is $2\varepsilon_B$.

In the case of two-vector annihilation the functions D_1 and D_2 are expressed through the form factors G_1, G_2, G_3 as follows:

$$D_1 = -\frac{P^2 K^2}{8M^2}\left|G_1 + \mu_B G_2 + \varepsilon_B G_3\right|^2,$$

$$D_2 = \frac{P^2}{4M^2}\left|G_1 + \mu_B G_2 + \varepsilon_B G_3\right|^2 - \left|G_1 + \frac{P^2}{2M^2}\varepsilon_B G_3\right|^2 - \frac{1}{2}\left|G_1 + \frac{P^2}{2M^2}\mu_B G_2\right|^2. \tag{12}$$

Substituting these D_1 and D_2 into (6) and (7), we obtain the cross section for two-photon annihilation of a polarized electron and positron.

The results obtained above show that, owing to the polarization of the initial electrons and positrons, the cross sections for two-particle annihilation may be substantially deformed in comparison with the cross section of the process for unpolarized particles. The possibility of the appearance of such deformations should be taken into account when carrying out experiments with colliding beams. At the same time, since the spins of the final particles are summed over, experiments with polarized initial particles give no new information about the form factors in comparison with experiments with unpolarized particles. This is seen directly from formulas (6) and (7), since the combinations of form factors D_1 and D_2 also enter into the cross section for unpolarized particles. In order to obtain new information about the form factors, it is necessary to measure the polarizations of the final particles.

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Note: Figure translations are in progress. See original paper for figures.