CURRENT CONVECTION OF THE POSITIVE COLUMN IN A MAGNETIC FIELD AT LOW PRESSURE

V. V. VLADIMIROV

1965

SovietRxiv

Abstract

Full Text

CURRENT CONVECTION OF THE POSITIVE COLUMN IN A MAGNETIC FIELD AT LOW PRESSURE

V. V. VLADIMIROV

(Presented by Academician M. A. Leontovich, March 18, 1965)

PHYSICS

1. In work [1] it was shown that in a high-pressure positive column, when the mean free paths of charged particles are considerably smaller than the characteristic dimensions of the system, current convection may develop, in whose mechanism collisions of charged particles with neutrals play an important role.

Under conditions in which the mean free paths and the Larmor radius of the ions are large ($l_i, l_e > a,\ r_{iH} > a$, where a is the tube radius), an interesting modification of current convection may appear, in whose mechanism factors associated with ion inertia play an important role. Among them the principal role is played by transport processes in the ambipolar electric field.

The dispersion equation for this case, when the axial current in the plasma is not taken into account, was obtained in work [2]. It is not difficult to generalize it to the case in which the drift velocity of the electrons v_0 is different from zero, by shifting the Maxwellian distribution of the electrons by the amount of the directed velocity.

2. The initial kinetic equations for electrons and ions have the form

$$\frac{\partial f_e}{\partial t} + \mathbf{v}\nabla f_e + \frac{e}{m_e}\nabla\varphi\,\frac{\partial f_e}{\partial \mathbf{v}} - [\mathbf{v}\cdot\vec{\omega}_{eH}]\,\frac{\partial f_e}{\partial \mathbf{v}} = Z\,\frac{n_e}{n_0}\,f_{0e}, \tag{1}$$

$$\frac{\partial f_i}{\partial t} + \mathrm{div}_r\,\mathbf{v} f_i - \frac{e}{M}\nabla\varphi\,\frac{\partial f_i}{\partial \mathbf{v}} = Z n_e \delta(\mathbf{v}), \tag{2}$$

$n_e = n_{e0} + n_e'$, where the subscript zero corresponds to an unperturbed quantity, and the prime denotes a perturbation of this quantity; $\omega_{eH} = eH/m_{ec}$; Z is the number of ionizations produced per unit time by one electron;

$$f_{0e} = \left(\frac{m_e}{2\pi T_e}\right)^{3/2} \exp\left\{-\frac{m_e}{2T_e}\left[v_\perp^2 + (v_z - v_0)^2\right]\right\}.$$

The ions are born with zero velocity upon ionization of neutral atoms by electrons, which is taken into account by the delta function.

Linearizing equations (1), (2) with respect to perturbations of the form

$$A'(r) = A'r\left(1-\frac{r^2}{a^2}\right)e^{-i\omega t+i\varphi+ik_z z} \qquad \text{(first mode)}$$

for parabolic profiles of the equilibrium density and radial potential

$$n(r) = n_0(1-Br^2), \qquad \varphi(r) = -Ar^2,$$

from the condition of quasineutrality ($n_i' = n_e'$) we obtain the dispersion equation [2], which in the approximation

$$\frac{\omega - k_{zv}0 + 2Ac/H}{k_z^2 v_{eT}} \gg 1 \tag{3}$$

has the form

$$\frac{\pi c}{8H}\,\frac{Ba^2(i\omega+Z)}{(\omega - k_z v_0 + 2Ac/H - iZ)}\left[1-\frac{k_z^2\omega_{eH}}{2B}\,\frac{(1-Ba^2/2)}{(\omega-k_z v_0+2Ac/H-iZ)}\right]-$$

$$-\frac{e}{M}q\left[\left({}^3/_{16}\,\pi\right)^2\frac{Ba^2}{1-{}^3/_{16}\,\pi i\omega q}-\left(\frac{\pi}{4}\right)^2\frac{(1+Ba^2)}{1-{}^1/_4\,\pi i\omega q}+\left(\frac{\pi}{2}\right)^2\frac{1}{1-{}^1/_2\,\pi i\omega q}+\right.$$

$$\left.+0.56\left(1-{}^3/_4 Ba^2\right)\right]=0, \qquad \text{where} \quad q=\left(\frac{2eA}{M}\right)^{-1/2}. \tag{4}$$

According to the solution of the Langmuir problem [2] on the equilibrium state of such a system, taking the magnetic field into account, $Z \approx 0.7/q$, and $Ba^2 \approx 1$ in a relatively weak magnetic field ~ 30 Oe.

For a small current velocity v_0, a weak magnetic field, and small $|k_z|$, such that

$$2Ac/H \gg \omega,\ |k_z|v_0, \tag{5}$$

the currentless excitation mechanism considered in [2] predominates. It is shown that in this case purely azimuthal waves with frequency $\omega \approx \frac{2}{a}\sqrt{\frac{e\varphi_0}{M}}$ can be excited, where φ_0 is the value of the ambipolar potential at the edge of the column. For $e\varphi_0 \sim T_e$, the phase velocity of these waves is, in order of magnitude, equal to the ion-sound velocity.

For a large current velocity, larger magnetic fields (but with $r_{iH} > a$), and larger k_z, when

$$|k_z|v_0 \gg 2Ac/H,\ \omega, \tag{6}$$

the current mechanism of instability may appear.

It is interesting to note that in equation (4) the term corresponding to the ambipolar drift of electrons ($2Ac/H$), which is responsible for the currentless excitation of instability, and the current term ($-k_{zv}0$) enter in exactly the same way for $k_z < 0$. Therefore one should expect that, under the current mechanism of instability, waves with $k_z < 0$ will be excited (this will be confirmed by the further calculation), which for the mode $m = 1$ corresponds to right-handed perturbations.

When condition (6) is satisfied, we shall investigate the dispersion equation (4) for the case

$$v_0 \gg |k_z|\omega_{eH}a^2/4, \tag{7}$$

when the second term in the first square bracket in equation (4) may be neglected.

At the stability boundary $\operatorname{Im}\omega = 0$, and equation (4) splits into two equations which, after simple transformations, reduce to the system:

$$\frac{0.29}{1+0.35x} - \frac{0.80}{1+0.62x} - \frac{0.34}{1+2.46x} + 0.2 = 0, \tag{8}$$

$$\frac{1}{k_{zv}0\omega_{iH}} + 2.55q^2\left(\frac{0.21}{1+0.35x} - \frac{0.98}{1+0.62x} + \frac{3.86}{1+2.46x}\right) = 0. \tag{9}$$

It is not difficult to show from equations (8), (9) that, under the condition

$$|k_z|v_0\omega_{iH} > 1.4/q^2 = 2.8\,e\varphi_0/Ma^2 \tag{10}$$

oscillations are excited with frequency

$$\omega_{\mathrm{cr}} \approx \frac{1.4}{q} \approx \frac{2}{a}\sqrt{\frac{e\varphi_0}{M}}. \tag{11}$$

As expected, waves with $k_z < 0$ are excited.

Thus, in both cases (5), (6), the oscillation frequency at the stability boundary is, in order of magnitude, the same as the inverse time of flight of an ion to the tube wall in the ambipolar electric field. For $e\varphi_0 \sim T_e$, the azimuthal component of the phase velocity of these waves coincides with the ion-sound velocity. We have called these waves [2] "ambipolar" sound, since they are generated only when transport in the ambipolar electric field is taken into account.

Assuming $e\varphi_0 \sim T_e$, $|k_z| \sim 1/L$, where L is the length of the discharge gap, we rewrite condition (10) in the form

$$\frac{a}{L}\frac{v_0}{\sqrt{T_e/M}} > \frac{r_i H}{a}. \tag{12}$$

For $L = 20$ cm, $a = 1$ cm, $T_e = 4$ eV, $M = 200$ atomic units (mercury), a potential difference along the column ≈ 2 V, $l_e > L$, this condition is satisfied for $H \gtrsim 120$ Oe.

It is possible that, in the experiments of Nedospasov et al. [3], carried out in short tubes with a mercury positive column, where the indicated conditions and approximations were fulfilled, current convection of the positive column, described in this note, took place.

In conclusion I express my deep gratitude to B. B. Kadomtsev, who made a number of valuable comments.

Received
16 III 1965

REFERENCES

[1] B. B. Kadomtsev, A. V. Nedospasov, *J. Nucl. Energy, Part C*, **1**, 230 (1960).
[2] V. V. Vladimirov, ZhETF, **48**, 175 (1965).
[3] L. L. Artsimovich, A. V. Nedospasov, S. S. Sobolev, *Nuclear Fusion*, **4**, 125 (1964).

Note: Figure translations are in progress. See original paper for figures.