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Abstract

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RAY FLUXES AND NATURAL OSCILLATIONS IN OPEN RESONATORS

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Geometrical optics is a powerful method for the theoretical study of all possible optical systems, provided that the wavelength is much smaller than all dimensions of the system. With the appearance of open resonators in the short-wavelength range, the problem arose of extending the methods of geometrical optics to them. This is hindered by the circumstance that such concepts as a natural wave function, a natural frequency, etc., are not inherent in geometrical optics. If geometrical optics is supplemented somewhat so that it includes these concepts, then an effective apparatus is obtained by means of which many problems of electrodynamics can be solved. Below we set forth an attempt to create such an apparatus.

The wave function characterizes the field distribution of one or another natural oscillation. On the other hand, it is known that a system of rays is bounded by caustic surfaces in such a way that the field is concentrated mainly in the region bounded by the mirrors and the caustic surfaces. Beyond a caustic surface the field decreases exponentially. Thus, from the point of view of geometrical optics, the field distribution is characterized simply by the position of the caustic surface. To determine the natural frequencies and the parameters of the caustic surface, quantum conditions are introduced. The number of quantum conditions is equal to the number of dimensions of the system.

In a given system of rays, the wave front may propagate along different paths. For a natural oscillation it is necessary that a wave front, having left some point, arrive at another point along different paths with a phase difference that is a multiple of 2π. In doing so, one must take into account the so-called phase jump at a caustic, equal to $\pi/2$, which does not follow directly from geometrical optics but whose existence is explained quite simply (see, for example, (1)).

Using similar considerations, in work (2) it proved possible to solve the two-dimensional problem of the natural oscillations of a mirror ellipse. It was shown that there exist two types of natural oscillations. In the three-dimensional case it is natural to consider the oscillations of a mirror ellipsoid.

From the theorems of geometry (3) it follows that a ray propagating inside a mirror ellipsoid and tangent to some surface confocal with the mirror ellipsoid, after reflection in the mirror ellipsoid will again touch the same surface. It is obvious that upon subsequent reflections the ray will touch the same surface of the confocal family. Consequently, this surface is the envelope of the family of rays, i.e., a caustic surface.

It can be shown that for a given straight line one can always construct two tangent surfaces from the given confocal family. Both of these surfaces are caustics and bound the region of space in which the wave field is mainly concentrated. Depending on the position of the ray, four different types of oscillations may arise.

We shall call the inner part of the plane xy the part bounded by the focal ellipse (4) and containing its foci. The remaining part of this plane will be called the outer part. Similarly, the part of the plane xz in which the foci of the focal hyperbola lie (consisting of two parts) will be called the inner part, and the remaining part the outer part.

Fig. 1
Fig. 1

Fig. 2
Fig. 2

Different types of oscillations arise depending on where the initial ray intersects the planes xy and xz. In this case, oscillations of four types are possible:

I. The initial ray intersects the outer parts of the planes xy and xz. The tangent surfaces to it will be an ellipsoid and a two-sheeted hyperboloid. The region filled with rays is schematically shown in Fig. 1; it has the form of a torus located near the plane yz, with a cross-section in the form of a curvilinear rectangle.

Fig. 3
Fig. 3

Fig. 4
Fig. 4

II. The initial ray intersects the outer part of the plane xy and the inner part of the plane xz. The tangent surfaces to it will be an ellipsoid and a one-sheeted hyperboloid. The region filled with rays has the form of a torus located near the plane xy (Fig. 2).

III. The initial ray intersects the inner part of the plane xy and the outer part of the plane xz. The tangent surfaces to it will be two-sheeted and one-sheeted hyperboloids. The region occupied by rays has the form of a curvilinear rectangular prism with planes of symmetry $x = 0$, $y = 0$, and $z = 0$ (Fig. 3).

IV. Finally, if the initial ray intersects the inner parts of the planes xy and xz, then the tangent surfaces to it will be two one-sheeted hyperboloids.

The region occupied by rays has the form of a hollow curvilinear cylinder elongated along the z-axis (Fig. 4).

Thus, Figs. 1–4 show possible types of oscillations in a mirror triaxial ellipsoid. Oscillations of types I and II have much in common with the so-called "whispering-gallery" waves ([5,6]), and this name may be extended to them. In open resonators formed by two spherical mirrors, oscillations of types III and IV are realized.

In order to pass from a mirror ellipsoid to an open resonator, it is necessary to take into account that beyond the caustic surface the wave field decreases exponentially. Therefore, if the mirror surface is cut off beyond the caustic surface at some distance from it, this will have little effect on the given oscillation. The meaning of the transition to an open resonator is to suppress the superfluous oscillations whose caustic surfaces are located differently with respect to the given mirror surface.

Let us now pass to the quantum conditions. On the caustic surfaces bounding a given oscillation there exist lines which, at each of their points, are tangent to rays belonging to the given oscillation. Such lines include, first of all, the lines of intersection of the caustic surfaces. Indeed, a straight line tangent to the line of intersection thereby touches both caustic surfaces and therefore belongs to the rays forming the oscillation, since after all subsequent reflections this straight line will still be tangent to the selected caustic surfaces. Geodesic lines on the caustic surfaces that touch the line of intersection of the two caustic surfaces have the same property.

It may be assumed that wave propagation along these lines occurs with a velocity equal to the speed of light in vacuum. The phase advance along the line of intersection upon return to the initial point must be a multiple of 2π. Thus, the first quantum condition is that the length of the line of intersection of the two caustic surfaces must be a multiple of the wavelength.

Similarly, the phase advance along a geodesic, beginning at its point of tangency with the line of intersection of the caustic surfaces and ending at a new point of tangency with the line of intersection, must differ by a term multiple of 2π from the phase advance along the line of intersection of the caustic surfaces between the same points. This gives us the second quantum condition. The third quantum condition is obtained from analogous reasoning, except that the geodesic should be chosen on the second caustic surface.

These arguments lead to three quantum conditions that determine the frequency of the natural oscillations and the position of the caustic surfaces corresponding to these oscillations. For oscillations of type I these quantum conditions have the form

$$k\int_0^{\theta_1} \sqrt{P(\xi)}\, d\xi = \left(2N_\xi + \frac{1}{2}\right)\pi, \qquad k\int_{c^2}^{b^2} \sqrt{P(\eta)}\, d\eta = N_\eta \pi,$$

$$k\int_{\theta_2}^{a^2}\sqrt{P(\zeta)}\,d\zeta=\left(N_\zeta+\frac{1}{2}\right)\pi, \tag{1}$$

where

$$P(\theta)=\frac{(\theta_1-\theta)(\theta_2-\theta)}{(a^2-\theta)(b^2-\theta)(c^2-\theta)};$$

ξ,η,ζ are ellipsoidal coordinates; a,b, and c are the semiaxes of the mirror ellipsoid; θ_1 and θ_2 are parameters determining the position of the caustic surfaces in the ellipsoidal coordinate system.

Similarly, an investigation of oscillations of type II, when the caustic surfaces are the ellipsoid $\xi=\theta_1$ and a one-sheeted hyperboloid

$\eta=\theta_2$, leads to the quantum conditions

$$k\int_0^{\theta_1}\sqrt{P(\xi)}\,d\xi=\left(2N_\xi+\frac{1}{2}\right)\pi,\qquad k\int_{c^2}^{\theta_2}\sqrt{P(\eta)}\,d\eta=\left(N_\eta+\frac{1}{2}\right)\pi, \tag{2}$$

$$k\int_{\theta^2}^{a^2}\sqrt{P(\zeta)}\,d\zeta=N_\zeta\pi.$$

For oscillations of type III (caustic surfaces $\eta=\theta_1$ and $\zeta=\theta_2$) the quantum conditions have the form

$$k\int_0^{c^2}\sqrt{P(\xi)}\,d\xi=N_\xi\pi,\qquad k\int_{\theta_1}^{b^2}\sqrt{P(\eta)}\,d\eta=\left(N_\eta+\frac{1}{2}\right)\pi,$$

$$k\int_{\theta_2}^{a^2}\sqrt{P(\zeta)}\,d\zeta=\left(N_\zeta+\frac{1}{2}\right)\pi. \tag{3}$$

Finally, for oscillations of type IV (caustic surfaces $\eta=\theta_1$ and $\eta=\theta_2$) the quantum conditions have the form

$$k\int_0^{c^2}\sqrt{P(\xi)}\,d\xi=N_\xi\pi,\qquad k\int_{\theta_1}^{\theta_2}\sqrt{P(\eta)}\,d\eta=(2N_\eta+1)\pi, \tag{4}$$

$$k\int_{b^2}^{a^2}\sqrt{P(\zeta)}\,d\zeta=N_\zeta\pi.$$

In all the quantum conditions given above, the numbers N_ξ, N_η, and N_ζ are regarded as integers $(0, 1, 2, ...)$. It should be remembered that at least one of them must be large. In the general case the quantum conditions are expressed in terms of hyperelliptic integrals. However, in particular cases, when the rays are paraxial or there is one or another symmetry, these integrals reduce to simple elliptic or elementary ones.

Many problems concerning the natural oscillations of open resonators can be reduced to the problem of a reflecting ellipsoid. At present we have thus solved the problems of oscillations between spherical mirrors, including the case in which plane-parallel plates inclined at Brewster' s angle are placed between the mirrors. A resonator with spherical mirrors of different curvature has also been considered. Many types of closed resonators with more than two mirrors have been studied, as well as open-type lens and mirror transmission lines.

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Note: Figure translations are in progress. See original paper for figures.