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Abstract

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GEOPHYSICS

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THE BASIC LAW OF DISTRIBUTION OF THE PARAMETERS OF NATURAL MAGNETIZATION IN IGNEOUS ROCKS

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The magnetic parameters χ and I_n, which characterize the natural magnetization of igneous rocks, are variable quantities that vary considerably within magmatic bodies. These variable quantities may be regarded as continuous. The nonuniformity in the distribution of the magnetization parameters reflects the physicochemical conditions that accompany the process of crystallization of an igneous rock (^{1}). The latter, for the rock as a whole, manifests itself differently in each small volume and therefore may be regarded as random (^{3}).

It is impossible to predict in advance what magnetization a specimen will possess as the result of a single measurement, and consequently the magnetic susceptibility χ and the remanent magnetization I_n may with full justification be considered random variables; in processing them one may apply the methods of variational statistics, which in its generalizations is based on the laws of probability theory. From probability theory it is known that a universal characteristic of a random variable is its distribution function. This function expresses the probability that the random variable ξ, as a result of a single observation, will assume a value less than or equal to a given x, i.e.

$$F(x) = \mathbf{P}(\xi \leq x).$$

The probability-density graphs, or variational curves, long used in the practice of geophysics are nothing other than a graphical representation of the distribution function.

Depending on the form of the distribution function of a random variable, it is determined by a different number of parameters. The normal distribution of a random variable is determined by the smallest number of parameters. The normal law is a very widespread model for distributions of many quantities

Figure 1

Figure 1: Figure 1

characterizing the physical properties of rocks. The normal distribution is symmetric and is completely determined by two parameters: the mean value of the random variable and its root-mean-square deviation.

Practical magnetic prospecting experience has long established that the distributions of χ and I_n in igneous rocks most often do not agree with the normal law, and the probability-density curves are asymmetric (see Fig. 1). For an analytical characterization of asymmetric distributions, at least four parameters are necessary: the arithmetic mean and the probable values, estimates of asymmetry and excess. In most cases, when constructing variational curves of χ and I_n, it is necessary to resort to a logarithmic scale. The boundaries of the grouping intervals then follow a geometric progression; very often the denominator of this progression (the logarithmic interval of the distribution) has to be taken equal to 10. All this led to the supposition that the empirical distributions of the quantities χ and I_n obey a logarithmic law. Will these distributions be logarithmically normal,

i.e., whether the logarithms of the magnetization quantities being studied will be distributed according to the normal law? This type of distribution, in comparison with other asymmetric distributions, has a number of advantages. First of all, a logarithmically normal distribution, like a normal one, can be fully characterized by just two parameters, μ and σ^2, whose statistical estimates are, respectively: the arithmetic mean of the logarithms of the quantities χ and I_n ($\overline{\lg \chi}$ and $\overline{\lg I_n}$), and also the variances of the logarithms of these quantities ($S^2_{\lg \chi}$ and $S^2_{\lg I_n}$). Cases of an assumed logarithmically normal distribution of magnetic

Fig. 1. Probability-density curves of magnetization parameters and of their logarithms (intrusion of granodiorites on the Kono River, Polar Urals, collection No. 18). a—magnetic susceptibility ($n = 98$; $\overline{\lg \chi} = 3.0137$; $S_{\lg \chi} = 0.128$); b—remanent magnetization ($n = 94$; $\overline{\lg I_n} = 2.3632$; $S^2_{\lg I_n} = 0.194$)

susceptibility and of the factor Q in igneous rocks have been noted in the literature ([2,4]), but no tests of the agreement of statistical distributions with the logarithmically normal law by means of statistical criteria have been carried out. The graphical methods used to check the conformity of an empirical distribution to one or another law are very approximate and may be used only for a rough, purely preliminary assessment of the type of distribution. Only analytical methods of testing, accepted in mathematical statistics, make it possible to draw justified conclusions about the type of distributions.

To test the hypothesis of a logarithmically normal distribution of the quantities χ and I_n, we used a reliable and verified method consisting in considering the quotients obtained by dividing the statistical estimate of the asymmetry of the distribution γ_1 by its standard deviation σ_{γ_1}, for a given number n of observations, and the statistical estimate of the excess γ_2 by the corresponding standard deviation σ_{γ_2} (^{3}).

The calculations were carried out according to the following formulas:

$$\overline{\gamma}_1 = \frac{1}{nS^3_{\lg x}} \sum_{i=1}^{n} (\lg x_i - \overline{\lg x})^3, \qquad \overline{\gamma}_2 = \left[\frac{1}{nS^4_{\lg x}} \sum_{i=1}^{n} (\lg x_i - \overline{\lg x})^4 \right] - 3,$$

where

$$\overline{\lg x} = \frac{1}{n} \sum_{i=1}^{n} \lg x_i, \qquad S_{\lg x} = \left[\frac{1}{n-1} \sum_{i=1}^{n} (\lg x_i - \overline{\lg x})^2 \right]^{1/2}.$$

The values of the corresponding standard deviations σ_{γ_1} and σ_{γ_2} were found from the approximate formulas:

$$\sigma_{\gamma_1} \simeq \sqrt{6/n}, \qquad \sigma_{\gamma_2} \simeq \sqrt{24/n}.$$

If the logarithms of the values χ and I_n are distributed entirely normally,

then the ratios $\overline{\gamma}_1/\sigma_{\gamma_1}$ and $\overline{\gamma}_2/\sigma_{\gamma_2}$ are asymptotically normally distributed with parameters $(0, 1)$. Therefore, if $|\overline{\gamma}_1/\sigma_{\gamma_1}| \leqslant 3$, $|\overline{\gamma}_2/\sigma_{\gamma_2}| \leqslant 3$, then the obtained values $\overline{\gamma}_1$ and $\overline{\gamma}_2$ should be regarded as random, arising as a result of the limited amount of available analytical data, and the distribution of the logarithms of the values χ or I_n as consistent with the normal law.

If, however, the ratios $\overline{\gamma}_1/\sigma_{\gamma_1}$ or $\overline{\gamma}_2/\sigma_{\gamma_2}$ exceed 3 in absolute value, then the values $\overline{\gamma}_1$ and $\overline{\gamma}_2$ cannot be neglected as random, and the distribution of the logarithms of the values χ or I_n should be regarded as not consistent with the normal law, while the distribution of the quantities χ and I_n themselves should be regarded as not consistent with the lognormal law (3). The results of testing the hypothesis of a lognormal distribution of the magnetization parameters are given in Table 1, where the numerator refers to magnetic susceptibility and the denominator to residual magnetization.

The test showed that the most widespread theoretical distribution function of the quantities χ and I_n in different types of igneous rocks, not contradicting the empirical distributions, is the lognormal function. Of 64 analytically tested distributions, only 11 are not consistent with the lognormal law. The predominantly lognormal distribution of the quantities χ and I_n in igneous rocks is in complete agreement with the fundamental law of geochemistry on the lognormal distribution of the contents of minerals and elements in igneous rocks ((3) and others), and once again attests to the direct and close connection between the magnetization of a rock and its content of ferrimagnetic component. The intensity and distribution of magnetization parameters reflect the content and nature of the distribution of accessory ferrimagnetic minerals. Therefore, apparently, it is in principle possible, on the basis of comparing the distributions of the quantities χ and I_n in rocks, to draw geochemical conclusions analogous to those that could be drawn on the basis of studying the distributions of the contents of accessory minerals. Thus, for example, it has been shown in geochemistry that deviations of the distributions of mineral contents from the lognormal law may be connected with the presence of superposed processes, expressed in substantial replacement of a given mineral by another, or in the formation of an independent, later generation of the same mineral (3). Everywhere where a deviation of the distributions of χ and I_n from the lognormal law is observed (collections Nos. 3, 5, 13, 20, 21), a strong effect of metamorphic processes is noted in the rocks, or at least two generations of ferrimagnetics are established. The discovered difference in the types of distribution of magnetic parameters for two groups of moderately acid granitoids (Polar Urals, collections Nos. 18 and 20) confirms the view that the indicated granitoids cannot be considered derivatives of a single gabbro-peridotite formation; these groups represent independent complexes, differing in formation affiliation and age. The predominantly lognormal distribution of the quantities χ and I_n makes logarithmic processing of these widely measured parameters of natural magnetization necessary.

Analytical methods for determining the type of distribution functions of the quantities χ and I_n must also be introduced into the practice of studies of the magnetic properties of rocks. Obviously, until the type of distribution of a parameter has been clarified, any statistical (magneto-geochemical) comparisons are impossible. The very type of distribution function of magnetic parameters characterizes the features of the formation of ferrimagnetics and, consequently, is a genetic indicator for the rock itself as well. Knowledge of the type of distribution function makes it possible to use

Table 1

Statistical characteristics and results of testing the logarithmically normal distribution of χ and I_n in igneous rocks and in their metamorphosed varieties (Urals)

Rocks	Locality or name of massif	n	$\overline{\lg x}$	$S^2_{\lg x}$	$\overline{\gamma_1}$	$\overline{\gamma_2}$	s_{γ_1}	σ_{γ_2}	$\frac{\overline{\gamma_1}}{\sigma_{\overline{\gamma_1}}}$	$\frac{\overline{\gamma}}{\sigma_{\overline{\gamma_2}}}$
Hyperbasites (enstatite dunites and saxonites)	Rai-Iz	106	3,14173,02352 0,715−5,550—0,1760+0,2410					0,476	2,2900,7381401,071	
Same	Syum-Key	60	3,35113,03916 0,668,4330−1,0280+0,3980					0,632	1,3640,322401,555	
Serpentinized dunite	M. Khadata	117	3,24622,07409 0,2,05451,8,12005,10226					0,452	2,4007,12,70011,400	
Hyperbasites (dunites and saxonites)	Khache-Ruz	43	3,31883,06316 0,600,995—0,5140+0,3760					0,750	2,4201,390530,350	
Peridotite	Mt. Maslo	9	3,49163,02034 0,1,30900,3,46000−0,1810					1,630	1,3400,425,09000,613	
Peridotites (harzburgites)	Kraka	51	2,63702,07840 0,5,38051,4,02580−0,6342					0,684	2,3601,2,40171,000	
Serpentinite	Elevation 918 m	29	4,08084,01,4132 0,3,34050,2,6560−1,3465					0,910	0,0120,5,28501,495	
Intrusive diabase	Mt. Yar-Key	19	3,07402,07438 1,1,06060,2,89,670−0,51,6370					1,126	2,8400,5,14821,040	
Hornblende gabbro	Mt. Maslo	49	3,36713,02871 0,5,21,6770−2,0850+0,1,5150					0,700	1,9300,5,95001,640	
Gabbro-norite	“”	29	3,59713,07,3614 0,3,3621,4,02400,7,8453					0,906	0,5753,08,2640,858	
Gabbro	Elev. 305 m	22	3,59662,08,4106 0,3,41201,5,30,9503,4,5222					1,044	0,2302,9,49503,280	
Pyroxenite	V. Kharuta	14	3,49073,03,3702 0,3,70,0630−6,01,020+0,7,7150					1,550	0,0810,7,7,5220,014	

Rocks	Locality or name of massif	n	$\overline{\lg x}$	$S^2_{\lg x}$	$\overline{\gamma_1}$	$\overline{\gamma_2}$	s_{γ_1}	σ_{γ_2}	$\frac{\overline{\gamma_1}}{\sigma_{\overline{\gamma_1}}}$	$\frac{\overline{\gamma}}{\sigma_{\overline{\gamma_2}}}$
Gabbro	"""	41	3,59493,1,13240,171,5582,1,50008,5,00404					0,808	2,8605,342001	0,500
Diorite	"""	24	3,45482,09,6240,2572680,033,670−0,5,170					1,022	0,5230,06457	0,657
Diorite	r. B. Khanmei	10	2,21362,03071,32970,351,350−0,774 0					1,548	0,9000,46372	1,130
Hybrid tonalite	Elev. 169 m	26	3,89103,02,1920,241,0200−1,9140−0,4880					0,960	2,1200,3145	0,083
Quartz diorite	Talbei-Shor brook	15	2,41291,042670,2,0360−0,012550−0,6330					1,260	0,5700,14230	1,290
Granodiorite	Kongor	9894	3,01372,03,62820,19,4040,17−5,240−0,2,1840,2,53960,5,06160,69,384							0,278
Granodiorite	Kharamatalou	13	3,27802,0,15411,0−87,977−1,42300,4,6180					1,360	1,4302,07,5 69	0,300
Tonalite	r. Sob –r. M. Khanmei	49	3,08392,01,62730,2−21,244+1,84073,0,7,9350					0,700	3,5503,02,5630	5,420
Spilite	r. Yun-Yakha	18	3,27862,06,38120,40−1,980−2,1,95012,08576					1,152	3,4303,12,4570	1,800
Effusive of basic composition	Bas. r. Shchuchya	23	3,25462,04,109640,2−71,290+1,0,8500,2,2509					1,020	2,5401,6,7657	0,218

use the most powerful statistical criteria and maximum-likelihood estimates when processing magnetization parameters.

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Note: Figure translations are in progress. See original paper for figures.