ON THE THEORY OF AEROSOL FILTRATION

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Abstract

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ON THE THEORY OF AEROSOL FILTRATION

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1°. The theory of diffusional deposition of aerosols on the fibers of an aerosol filter has until now been based either on the model of an isolated circular cylinder, washed by a flow normal to its axis under laminar conditions ([1,2]), or on the more exact model of parallel cylinders under the condition that the distance $2b$ between their axes is large in comparison with the cylinder diameter $2a$ ([3]). In the latter case the pressure gradient averaged over the thickness of the filter is determined directly from the results ([4,5]) of approximate calculations of the flow field in a system of randomly arranged parallel cylinders and has the form

$$\partial p/\partial x = 4\mu U a^{-2}\sigma(-{}^{1}/_{2}\ln\sigma - \lambda)^{-1}, \qquad (1)$$

where μ is the viscosity of the liquid; U is the flow velocity before the filter; $\sigma = a^2/b^2$; λ is a numerical coefficient whose value, according to Happel' s calculation ([4]), is 0.5, and according to Kuwabara' s calculation ([5]), $\lambda = 0.75$. In this case the capture coefficient of aerosol particles, for a diffusion boundary layer small in thickness in comparison with the radius of the cylinder (i.e., under the condition that the Peclet number $\mathrm{Pe} = 2Ua/D \gg 1$, where D is the diffusion coefficient of the particles), and with neglect of inertia and interception effects (the latter condition corresponds to the requirement $(R/a)^2\mathrm{Pe}^{2/3} \ll 1$, where R is the particle radius), is determined directly from the results of works ([1,4,5]) and is expressed in the form ([3])

$$E = 2.9\mathrm{Pe}^{-2/3}(-{}^{1}/_{2}\ln\sigma - \lambda)^{-1/3}. \qquad (2)$$

Formulas (1) and (2) were obtained under the assumption that $\sigma^{1/2} \ll 1$, and already at $\sigma \approx 0.37$ (for $\lambda = 0.5$) or $\sigma \approx 0.22$ ($\lambda = 0.75$) they give infinitely large values of $\partial p/\partial x$ and E.

In the present note we consider the case when the distance between the surfaces of neighboring fibers is small in comparison with the fiber diameter, i.e. $\sigma^{1/2} \sim 1$,

Fig. 1

Figure 1: Fig. 1

and obtain the corresponding expressions for the pressure gradient averaged over the thickness of the filter and for the capture coefficient.

2°. Let us consider the motion of a liquid in a narrow gap between two parallel cylinders whose axes are normal to the direction of the flow. By methods of the hydrodynamic theory of lubrication* it is not difficult to show that, at small Reynolds numbers based on the width of the gap, the distribution of the longitudinal component of velocity in the coordinate system shown in Fig. 1 (region II) is determined in the form $v_x = -{}^1/_2\mu^{-1}(h^2 - y^2)\,\partial p/\partial x$, where $h = h(x)$ is the local half-width of the gap; $\partial p/\partial x$ is the longitudinal component of the pressure gradient, depending only on x. Since the liquid flow rate in the gap per unit length of cylinder is constant and equal to $q = -{}^2/_3\mu^{-1}h^3\partial p/\partial x$, for the longitudinal component of the velocity v_x and the pressure drop Δp between the sections $x = x_1$ and $x = x_2$ (inside the gap) we have

$$v_x = {}^3/_4 q\,(h^2 - y^2)h^{-3}, \qquad \Delta p = {}^3/_2\mu q \int_{x_1}^{x_2} h^{-3}\,dx. \tag{3}$$

* The possibility of such a treatment was noted, for example, in work ([6]).

We note that, with the corresponding definition of the function $h = h(x)$, formulas (3) are valid for cylinders of any cross section, provided that the half-width of the gap is small in comparison with the radius of curvature; for circular cylinders $h = b - (a^2 - x^2)^{1/2}$, where x is the distance along the gap axis from the minimum cross section.

In the case of a filter consisting of a system of parallel circular cylinders arranged in a staggered order, when the axis of each cylinder passes through the center of a regular hexagon at whose vertices lie the axes of neighboring cylinders, so that the distances between them are equal to $2b$ (Fig. 1), the liquid passes through a system of geometrically similar "horizontal" (type II) and "inclined" (types I, III) channels. By symmetry it is obvious that the flow rates of liquid in channels of types I, III are equal to half the flow rate in a channel of type II, the latter being equal to $q = 2bU$ per unit length of cylinder. Thus, formulas (3) for determining the longitudinal component of velocity and the pressure drop in channels of type II determine the values of v_x and Δp also for channels of types I, III, if q is replaced by ${}^1/_2 q$ and the corresponding coordinate transformation is performed.

Fig. 1

Taking into account that the abscissas of the points α', α'' and β', β'' are respectively $-a/2$ and $a/2$, we determine the pressure drop over the entire channel II

from the second formula (3) at $x_1 = -a/2$, $x_2 = a/2$. It is not difficult to see that then the pressure gradient averaged over the filter thickness, determined from the head losses in channels I, II, III, is equal to

$$\overline{\partial p}/\partial x = 3\mu U a^{-2}\sigma^{5/2}(1-\sigma)^{-2}\left\{1+\frac{3}{4}\sqrt{3}\sigma^{1/2}+2\sigma\left(1-\frac{1}{2}\sqrt{3}\sigma^{1/2}\right)^{-1}-\frac{1}{16}\sqrt{3}\sigma^{3/2}\left(1-\frac{1}{2}\sqrt{3}\sigma^{1/2}\right)^{-2}+\right.$$

$$\left.+6\sigma^{1/2}(1-\sigma)^{-1/2}\operatorname{arctg}\left[(2-\sqrt{3})(1+\sigma^{1/2})(1-\sigma)^{-1/2}\right]\right\}. \qquad (4)$$

3°. Bearing in mind the assumptions stated in item 1° and the expression for the longitudinal component of velocity according to item 2°, let us consider the diffusion process onto the lower half of the cylinder shown in Fig. 1.

The equation of stationary convective diffusion, neglecting the velocity of diffusive transport in the transverse direction in comparison with the velocity of radial transport, has the form

$$v_r\frac{\partial c}{\partial r}+\frac{v_\theta}{r}\frac{\partial c}{\partial \theta}=D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c}{\partial r}\right).$$

Following V. G. Levich (7), we pass from the coordinates (r, θ) to the coordinates (ψ, θ), where ψ is the stream function connected with the velocity components v_θ and v_r by the relations $v_\theta = \partial\psi/\partial r$, $v_r = -r^{-1}\partial\psi/\partial\theta$. We obtain

$$\frac{\partial c}{\partial \theta}=D\frac{\partial}{\partial \psi}\left(r\frac{\partial \psi}{\partial r}\frac{\partial c}{\partial \psi}\right). \qquad (5)$$

We define the stream function ψ according to (3) and the incompressibility condition in such a way that on the surface of the semicylinder under consideration $\psi = 0$. We obtain for region II

$$\psi=\frac{3}{4}q(r-a)^2\sin^{-2}\theta\,(b-a\sin\theta)^{-2}+O((r-a)^3),$$

$$\pi/3 \le \theta < 2\pi/3.$$

In view of what was said in item 2°, the stream function near the entire surface of the semicylinder can be defined by the expression

$$\psi = {}^3/_4 q(r-a)^2\chi^2(\theta)+O((r-a)^3),$$

where

$$\chi(\theta) = \begin{cases} 2^{-1/2}\{\sin(\theta+\pi/3)[b-a\sin(\theta+\pi/3)]\}^{-1}, & 0 \le \theta < \pi/3, \\ \{\sin\theta[b-a\sin\theta]\}^{-1}, & \pi/3 \le \theta < 2\pi/3, \\ 2^{-1/2}\{\sin(\theta-\pi/3)[b-a\sin(\theta-\pi/3)]\}^{-1}, & 2\pi/3 \le \theta \le \pi. \end{cases} \tag{6}$$

Then

$$\partial\psi/\partial r = (3q)^{1/2}\psi^{1/2}\chi(\theta) + O(\psi). \tag{7}$$

Let us consider, as usual, the case in which the thickness of the diffusion boundary layer is small in comparison with the width of the gap. Restricting ourselves to the first terms of the expansions in powers of $(r-a)$, we reduce the diffusion equation (5), taking (7) into account, to the form

$$\frac{\partial c}{\partial \theta} = Da(3q)^{1/2}\chi(\theta)\frac{\partial}{\partial\psi}\left(\psi^{1/2}\frac{\partial c}{\partial\psi}\right). \tag{8}$$

To integrate equation (8) we shall use the method of automodel solutions, developed by V. G. Levich (7) for problems of a similar type. Introduce the variable

$$\eta = \psi^{1/2}\left[aD(3q)^{1/2}\int_{\theta_1}^{\theta}\chi(\theta)\,d\theta\right]^{-1/3}, \tag{9}$$

as a result of which (8) is reduced to an ordinary differential equation, whose solution under the boundary conditions

$$c = 0 \quad \text{at } \psi = 0; \qquad c \to c_0 \quad \text{as } \psi \to \infty$$

has the form

$$c = c_0({}^4/_9)^{1/3}[\Gamma({}^4/_3)]^{-1}\int_0^{\eta}\exp(-{}^4/_9\eta^3)\,d\eta. \tag{10}$$

In determining the constant θ_1 in formula (9) we shall proceed from the following considerations. At the point of incidence of the flow at the entrance into channel I ($\theta = 0$), for values of ψ arbitrarily little different from zero, one must have $c = c_0$; this gives $\theta_1 = 0$ for region I. Further, if the flows entering channel II from channel I and from the channel symmetric to it with respect to the x-axis do not mix, then the boundary layer on the surface under consideration in channel II joins the boundary layer in channel I, the concentration c decreases smoothly along the streamline, and $\theta_1 = 0$ also for region II. In an analogous manner the boundary layers join when passing from channel II into channel

III. One may imagine, however, another limiting case, when complete mixing occurs upon the merging of the flows entering channel II (i.e., complete mixing in the wake region of each cylinder). In this case the boundary layers in channels I and II do not join, and the separation point α' (see Fig. 1), corresponding to the value $\theta = \pi/3$, should be regarded as a new point of incidence of the flow and one should set $\theta_1 = \pi/3$ for the region $\pi/3 \leq \theta \leq \pi$ (bearing in mind that the boundary layers in channels II and III still join). Thus, in the absence of mixing in the wake region (case A), $\theta_1 = 0$ for $0 \leq \theta \leq \pi$, while with complete mixing in the wake region (case B), $\theta_1 = 0$ for $0 \leq \theta < \pi/3$ and $\theta_1 = \pi/3$ for $\pi/3 \leq \theta \leq \pi$.

The diffusion flux to the surface under consideration is determined in the form

$$j = D\left(\frac{\partial c}{\partial r}\right)_{r=a} = 3^{-1/3}[\Gamma({}^4/_3)]^{-1} c_0 (Ua^{-1}bD^2)^{1/3} \chi(\theta) \left\{ \int_{\theta_1}^{\theta} \chi(\theta)\, d\theta \right\}^{-1/3} . \tag{11}$$

Then, taking into account what was said above concerning the values of θ_1, we obtain for the total diffusion flux over the entire surface of the half-cylinder under consideration

$$J = \int_0^{\pi} a(j)_{\theta_1=0}\, d\theta \qquad \text{in case A,}$$

$$J = \int_0^{\pi/3} a(j)_{\theta_1=0}\, d\theta + \int_{\pi/3}^{\pi} a(j)_{\theta_1=\pi/3}\, d\theta \qquad \text{in case B,}$$

and, according to (6) and (11), the capture coefficient is equal to

$$E = \frac{J}{Uac_0} = \frac{9}{2\Gamma(4/3)}\,\mathrm{Pe}^{-2/3}\left(\frac{a}{b}\right)^{1/3} \left\{ \int_{\pi/3}^{2\pi/3} \frac{d\theta}{\sin\theta\,(1 - \frac{a}{b}\sin\theta)} \right\}^{2/3} \times$$

$$\times \begin{cases} (1 + 2^{1/2})^{2/3}, & \text{(A)} \\ \left[2^{-1/3} + (1 + 2^{-1/2})^{2/3}\right], & \text{(B),} \end{cases}$$

or

$$E = \mathrm{Pe}^{-2/3} \delta^{1/6} \left\{ \ln 3 + 2\left(\frac{\sigma}{1-\sigma}\right)^{1/2} \operatorname{arcctg} \frac{3^{1/2} - 2\sigma^{1/2}}{(1-\sigma)^{1/2}} \right\}^{2/3} \times \begin{cases} 3,33, & \text{(A)} \\ 4,11, & \text{(B).} \end{cases} \tag{12}$$

Fig. 2

Figure 2: Fig. 2

Thus, the capture coefficients in the limiting cases of absence of mixing and of complete mixing in the wake region of the cylinder differ only slightly by a numerical factor; if the mean value of this factor is adopted, then the indicated limiting cases will correspond to corrections of −10 and +10%.

Fig. 2

The values $E\,\mathrm{Pe}^{2/3}$ for both limiting cases according to (12) are presented in Fig. 2 (curves A and B, respectively) as functions of the coefficient of filling of the space by the filter fibers α (for a staggered arrangement $\alpha = (\pi\sqrt{3}/6)\sigma$). For comparison, the dotted curve corresponding to dependence (2), for the case $\alpha = \sigma \ll 1$ (with $\lambda = 0.5$), is also shown here. In the upper left corner of Fig. 2 the schemes of the diffusion boundary layers in cases A and B are also shown, with the layer thickness $\delta \sim j/Dc_0$ calculated by formula (11).

Dependence (12) is valid under the restrictions indicated in item 1^0, imposed on the values of the Péclet number and the relative sizes of the aerosol particles. In addition, the lengths of the entrance sections in the gaps $L \sim \mathrm{Re}(b-a)$, where the Reynolds number $\mathrm{Re} = U(b-a)/\nu$, must be small in comparison with the channel length a, i.e. $\mathrm{Re} \ll a/(b-a)$. The latter also applies to dependence (4).

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Note: Figure translations are in progress. See original paper for figures.