ON THE CALCULATION OF SPATIAL SUPERSONIC GAS FLOWS BY THE METHOD OF CHARACTERISTICS

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Abstract

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ON THE CALCULATION OF SPATIAL SUPERSONIC GAS FLOWS BY THE METHOD OF CHARACTERISTICS

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The paper considers spatial steady supersonic motions of a gas with a detached shock wave.

In the space xyz there is given a blunt body Γ with a specified and continuously varying direction of the normal (Fig. 1). A uniform supersonic flow impinges on this body. We write the system of equations for determining the flow parameters in the region between the shock wave and the body in the form (1)

$$\operatorname{rot} \mathbf{u} \times \mathbf{u} - a^2 \nabla \mathcal{E} + \nabla I = 0; \tag{1}$$

$$\mathbf{u} \cdot \nabla (\mathbf{u}^2/2) - a^2 \operatorname{div} \mathbf{u} = 0; \tag{2}$$

$$\mathbf{u} \cdot \nabla \mathcal{E} = 0; \tag{3}$$

$$a^2 = \frac{\chi - 1}{2}(1 - \mathbf{u}^2); \tag{4}$$

$$\mathcal{E} = \frac{1}{\chi(\chi - 1)} \ln \frac{p}{\rho^\chi}; \tag{5}$$

$$I = \frac{\mathbf{u}^2}{2} + \frac{\chi}{\chi - 1} \frac{p}{\rho}; \tag{6}$$

$\mathbf{u}$ is the gas velocity referred to the maximum velocity w_m; ρ is the ratio of the gas density to the density of the oncoming flow ρ_∞; p is the pressure referred to the quantity $\rho_\infty w_m^2$. The coordinates in (1)–(3) are referred to the quantity r_0, which in each particular problem is chosen in an appropriate manner.

Fig. 1

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 2

We shall assume $I = \text{const}$ throughout the entire flow field. In this case one considers the system of equations (1)–(2), which for $|\mathbf{u}| > a$ has real characteristic surfaces on which characteristic relations (1) are satisfied. As the basis for calculating an individual point, the scheme proposed in (1), with some additions, is adopted. An iterative scheme for calculating an individual point is proposed.

Let $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$, $M_3(x_3, y_3, z_3)$ be given points (Fig. 2), and let $\mathbf{u}_1, \vartheta_1$; $\mathbf{u}_2, \vartheta_2$; $\mathbf{u}_3, \vartheta_3$ be the values of $\mathbf{u}$ and ϑ at the points M_1, M_2, M_3, respectively. On the triangle $M_1M_2M_3$ a tetrahedron is constructed according to a certain rule. It is required to find the coordinates of the vertex S of the tetrahedron and the values of $\mathbf{u}$ and ϑ at it. The face of the tetrahedron is constructed from the values of $\mathbf{u}$ at the midpoints of $M_{iM}k,\ i \neq k;\ i, k = 1, 2, 3,$ which can be obtained by linear interpolation:

$$\mathbf{u}_1^{(0)} = {}^1/_2(\mathbf{u}_1 + \mathbf{u}_2); \qquad \mathbf{u}_2^{(0)} = {}^1/_2(\mathbf{u}_2 + \mathbf{u}_3); \qquad \mathbf{u}_3^{(0)} = {}^1/_2(\mathbf{u}_1 + \mathbf{u}_3).$$

Having constructed an elementary characteristic tetrahedron from these velocity values, we compute the coordinates of the point S and, using the characteristic relations on the faces, find the values of $\mathbf{u}$ and ϑ at this point in the first approximation. We obtain $S(x_4^{(1)}, y_4^{(1)}, z_4^{(1)})$, $\mathbf{u}_4^{(1)}, \vartheta_4^{(1)}$.

The subsequent values $x_4^{(j)}, y_4^{(j)}, z_4^{(j)}, \mathbf{u}_4^{(j)}, \vartheta_4^{(j)}$ will be computed according to the following iteration scheme:

$$\mathbf{u}_1^{(j)} = {}^1/_2\left[{}^1/_2(\mathbf{u}_1 + \mathbf{u}_2) + \mathbf{u}_4^{(j)}\right]; \qquad \mathbf{u}_2^{(j)} = {}^1/_2\left[{}^1/_2(\mathbf{u}_2 + \mathbf{u}_3) + \mathbf{u}_4^{(j)}\right];$$

$$\mathbf{u}_3^{(j)} = {}^1/_2\left[{}^1/_2(\mathbf{u}_1 + \mathbf{u}_3) + \mathbf{u}_4^{(j)}\right].$$

From these velocity values the faces of the elementary tetrahedron are constructed, and the computation of the coordinates of the point S and of the parameters $\mathbf{u}$ and ϑ at it is carried out according to the same scheme. Thus the computation of an individual point will proceed. For proper inclusion of the points found in the subsequent computation, a method based on the use of matrices is proposed.

On the body Γ (Fig. 1) we choose a line L in such a way that the tangent vector to it at any point lies outside the characteristic cone drawn from that point. In this case two characteristic surfaces π_1 and π_2 pass through L. Suppose that the characteristic surface π passing through L, and the distribution of $\mathbf{u}$ and ϑ on it, are known to us. On the line L we choose n points, take one of them as the initial one, and write a matrix whose first row consists of the coordinates of this point and the values of $\mathbf{u}$ and ϑ at it. The second row of the matrix consists of the coordinates of the next division point and the values of $\mathbf{u}$ and ϑ at this point. The matrix has n rows:

$$A_1 \sim \begin{pmatrix} \mathbf{r}_{11} & \mathbf{u}_{11} & \vartheta_{11} \\ \mathbf{r}_{12} & \mathbf{u}_{12} & \vartheta_{12} \\ \dots & \dots & \dots \\ \mathbf{r}_{1n} & \mathbf{u}_{1n} & \vartheta_{1n} \end{pmatrix}.$$

Next, at some distance from L on the surface π, the next row of n points is taken. Similarly the matrix A_2 is formed:

$$A_2 \sim \begin{pmatrix} \mathbf{r}_{21} & \mathbf{u}_{21} & \vartheta_{21} \\ \mathbf{r}_{22} & \mathbf{u}_{22} & \vartheta_{22} \\ \dots & \dots & \dots \\ \mathbf{r}_{2n} & \mathbf{u}_{2n} & \vartheta_{2n} \end{pmatrix}$$

and so on. The last row of points belongs to the shock wave:

$$A_m \sim \begin{pmatrix} \mathbf{r}_{m1} & \mathbf{u}_{m1} & \vartheta_{m1} \\ \mathbf{r}_{m2} & \mathbf{u}_{m2} & \vartheta_{m2} \\ \dots & \dots & \dots \\ \mathbf{r}_{mn} & \mathbf{u}_{mn} & \vartheta_{mn} \end{pmatrix}.$$

To each matrix A_k we assign a point on the plane. Depending on the type of π, the correspondence is established in different ways. Two basic problems can be formulated.

Problem 1. The surface π is of type π_2. It is required to determine $\mathbf{u}$ and ϑ in the region enclosed between the body Γ, the characteristic surface π, and the characteristic surface of type π_2 passing through the line of intersection of π and the shock wave. The correspondence between the matrices and the points is established as shown in Fig. 3a. The order of choice of the matrices

for the computation is the same as the order of selecting the corresponding points for computation in the two-dimensional problem with data on a characteristic of the first family in the case when there is a solid wall.

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

Problem 2. The surface π is of type π_2. It is required to determine u and $\mathcal{E}$ in the region bounded by the shock wave χ, the characteristic surface π, and the characteristic surface of type π_2 passing through L. The shape of the shock wave is not known in advance and is found in the course of solving the problem. The correspondence between the initial matrices and the points in the plane is shown in Fig. 3b. The order of selecting the matrices in this problem is the same as the order of selecting the corresponding points for computation in the two-dimensional problem with data on a characteristic of the second family and a solid wall.

Fig. 3

Thus, a logical analogy has been obtained with the solution of the corresponding two-dimensional problems, which considerably facilitates the formulation of spatial non-axisymmetric problems for computation on high-speed electronic computers.

Fig. 4. $M_\infty = 4$. a $-\alpha = 0$, $\theta = 10^\circ$, black dots $-$data (3); $-\alpha = 1^\circ$, $\theta = 10^\circ$; $-\alpha = 5^\circ$, $\theta = 9.5^\circ$; $-\alpha = 10^\circ$, $\theta = 9.5^\circ$. 1 $-\varphi = 0$; 2 $-\varphi = \pi$

As an example, the flow past a cone blunted by a sphere and arbitrarily positioned relative to the oncoming stream was computed for $M_\infty = 4$. New schemes were developed for computing the points of the shock wave and the body. In computing the shock wave, exact relations on the wave were used; the characteristic relations were satisfied with a prescribed accuracy ε. Variation of ε over wide limits (10^{-3}—10^{-7}) has little effect on the shape of the shock wave. In computing the points of the body, the characteristic condition along the flow plane was used.

Figure 4 shows the pressure distribution along the generators of the cone. In the case $\alpha \neq 0$, the pressure distribution is given in the planes of symmetry $\varphi = 0$, $\varphi = \pi$ ($\varphi = 0$—windward side, $\varphi = \pi$—leeward side). R is the radius of blunting of the cone, x is directed along the cone axis (the origin is at the blunting), θ is the cone semi-vertex angle, α is the angle of attack, and p_∞ is the pressure in the oncoming flow.

The method makes it possible to calculate the surface of a weak discontinuity. Calculations show that the coordinate derivatives of the flow parameters undergo a discontinuity on passing through the characteristic surface that includes the line of junction of the sphere and cone. This discontinuity weakens with distance from the junction line, which agrees with the general physical picture of the flow. The present method can also be applied to other problems of gas dynamics.

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Note: Figure translations are in progress. See original paper for figures.