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ON THE SYMMETRY OF ELEMENTARY PARTICLES

IN THE THEORY OF A NONUNIFORM GROUP

(Presented by Academician V. A. Fock on February 10, 1965)

1. Recently, searches for nontrivial extensions of the Poincaré group have attracted attention. Extending the Poincaré group so as to include in a unified treatment both space-time variables and variables of the internal group is a natural development of theories in which the full group was written as the product of two groups (the Poincaré group and the group of internal symmetry). A drawback of theories using the direct product of groups is the circumstance that internal symmetries must be regarded in them only as a rough approximation to a future exact theory. The necessity of introducing, in addition, a special mechanism that violates symmetry makes all theories based on the direct product of groups purely phenomenological, since the mechanism of symmetry violation is usually connected with unknown dynamical effects.

On the basis of the success of the phenomenological description of symmetry by means of the group SU_6, we have constructed an extended group $P(SU_6)$ ([1]), for which the group SU_6 is the little group and which goes over in the static limit into SU_6. Groups of an analogous type (i.e., containing SU_6 as a subgroup) have been considered in various aspects in works ([2–6]). In contrast to theories using the direct product of groups, in our theory the assumption of the exact character of the symmetry proves to be noncontradictory, since the group contains within itself also the mechanism of symmetry breaking (in the former sense); we are free of restrictions connected, for example, with the equality of masses in supermultiplets. In the group $P(SU_6)$ under consideration, the energy-momentum vector belongs to the number of generators of the group, and therefore the concept of exact symmetry here already includes symmetry breaking in the former sense.

2. One should distinguish the group $P(SU_6)$ without reflections and the group $\bar{P}(SU_6)$, which contains reflections. Let us first consider the group $P(SU_6)$. The commutation relations between the 35 generators of "rotations" M_a, the 35 generators of "Lorentz" rotations N_a, and the 36 momenta P_α may be written in the form

$$[M_a, M_b] = iF_{abc}M_c, \qquad [N_a, N_b] = iF_{abc}M_c, \qquad [M_a, N_b] = iF_{abc}N_c, \tag{1}$$

$$[P_\alpha, M_b] = iF_{\alpha bc}P_c, \qquad [P_\alpha, N_b] = D_{\alpha bc}P_c, \qquad [P_\alpha, P_b] = 0. \tag{2}$$

Here the Latin indices run through the values $1, \dots, 35$, and the Greek ones $0, 1, \dots, 35$; F_{abc} are the structure constants of the group SU_6. If one constructs 36 linearly independent complex Hermitian matrices A_α, having 6 rows and columns, then

$$F_{abc} = \frac{1}{n}\,\mathrm{Sp}(A_a[A_b, A_c]), \qquad D_{\alpha\beta\sigma} = \frac{1}{n}\,\mathrm{Sp}(A_\alpha\{A_\beta, A_\sigma\}). \tag{3}$$

Here $\mathrm{Sp}\,A_\alpha A_\beta = n\delta_{\alpha\beta}$, $A_0 \sim 1$.

Let us form the form $\hat{P} = A_0P_0 + A_{aP}a$. Under an arbitrary transformation $U, \det U = 1$, the matrix $\hat{P}$ is transformed as $\hat{P}' = U\hat{P}U^+$. The invariant of this transformation is the determinant of the form $\hat{P}$: $Z = \det \hat{P} = \text{inv}$. The quantity Z plays for the group $P(SU_6)$ the same role as the invariant $p_0^2 - p^2$ for the Poincaré group (^{1}). Let us note, however, the fact that in the case of the Poincaré group this invariant does not depend on whether reflections of the spatial coordinates are included in the group. In the case of the group $P(SU_6)$, the invariant Z changes under reflection.

3. Let us compute the quantity Z. For this purpose we shall represent the determinant in the form of a homogeneous polynomial of the 6th degree, containing the momenta P_a by means of invariants of the group SU_6, constructed from the generators M_a. In this group there are 5 independent invariants, composed from the momenta P_a, which may be chosen in the form

$$\begin{aligned} y_2 &= P_{aP}a, & y_4 &= D_{abc}P_{aP_{bV}}c \equiv P_{aW}a, \\ y_3 &= D_{abc}P_{aP_{bP}}c \equiv P_{aV}a, & y_5 &= D_{abc}P_{aP_{bW}}c \equiv P_{aR}a \quad \text{etc.} \end{aligned} \tag{4}$$

The condition of invariance of Z with respect to "Lorentz" transformations associated with the generators N_a has the form

$$(P_a\,\partial/\partial P_0 + P_0\,\partial/\partial P_a + D_{abc}P_b\,\partial/\partial P_c)Z = 0. \tag{5}$$

From this we find

$$Z = P_0^6 - 3P_0^4 y_2 + 2P_0^3 y_3 + 3P_0^2(y_2^2 - \tfrac{1}{2}y_4) + \tfrac{6}{5}P_0(y_5 + y_2 y_3) - \\ - y_6 + y_3^2 - y_2^3 + \tfrac{5}{2}y_2 y_4. \tag{6}$$

Let us give the relations between the quantities $y_7, y_8, ...$, composed according to rule (4) from the momenta P_a:

$$\begin{gathered} y_7 = \tfrac{11}{5}y_2 y_5 + \tfrac{3}{5}y_3 y_4 - \tfrac{4}{5}y_2^2 y_3, \\ y_8 = 2y_2 y_6 + \tfrac{1}{2}y_4^2 - y_2^2 y_4 + \tfrac{7}{10}y_3 y_5 + \tfrac{1}{5}y_2 y_3^2, \\ y_9 = \tfrac{7}{10}y_4 y_5 + \tfrac{17}{5}y_2^2 y_5 + \tfrac{16}{5}y_2 y_3 y_4 + y_3 y_6 - \tfrac{8}{5}y_2^3 y_3. \end{gathered} \tag{7}$$

In the group $P(SU_6)$ the determinant (6) is the unique invariant which can be constructed from a single vector P_a.

Let us now introduce a "polarization" vector, which serves as the analogue of the polarization vector of the Poincaré group. This vector w_a is expressed in terms of the momenta and the rotation generators:

$$w_a = [D_{a\beta\sigma} P_\beta M_\sigma + iF_{\beta\alpha\sigma} P_\sigma N_\beta]. \tag{8}$$

It is not difficult to verify that w_a commutes with the momenta P_β. If the symmetry is excluded, leaving only the generators of the Poincaré group, then w_a becomes the ordinary polarization vector. But w^2 does not commute with the generators N_a.

4. For representations of the group $P(SU_6)$ one cannot construct an invariant bilinear Hermitian form. It is therefore necessary to pass to the extended group $\bar{P}(SU_6)$, which includes reflections. In the group $\bar{P}(SU_6)$ there are 72 momenta P_a and $\tilde{P}_a$ with invariant form

$$P_0^2 - P_{aP}a - \tilde{P}_0^2 + \tilde{P}_a \tilde{P}_a = m^2. \tag{9}$$

If one introduces the operators $P^\pm = P \pm \tilde{P}$, then the commutation relations for the momenta split into two groups (^{1}).

$$\begin{gathered} [P_a^+, N_b] = D_{abc} P_c^+, \qquad [P_\alpha^+, P_\beta^-] = 0, \\ [P_a^-, N_b] = -D_{abc} P_c^- + D_{ab0} P_0^-, \\ [P_a^\pm, M_b] = iF_{abc} P_c^\pm. \end{gathered} \tag{10}$$

In other words, the vector $B_a = -P_a^-$, $B_0 = P_0^-$ is transformed according to the conjugate representation, which has the opposite sign before the generator N_a:

$$B' = (1 + iM_a\alpha_a + N_a\beta_a)B. \tag{11}$$

Here, as before, $(N_a)_{\lambda\sigma} = D_{a\lambda\sigma}$ in the regular representation.

Under reflection, $IP_a^+I^{-1} = -P_a^- = B_a$, $IP_0^+I^{-1} = P_0^- = B_0$. Reflection can be represented by the matrix ρ_1, acting on a column composed of the representations (n, m) and (m, n) of the group $\bar{L}(SU_6)$, i.e., in the present case on a column made up of P^+ and B.

Let us write the invariant equation of motion for an arbitrary representation, which plays the role of the Klein–Fock equation. Since the only invariant of the group $P(SU_6)$ composed of momenta is the quantity $\det \hat{P} = Z$, then, taking reflections into account, the equation of motion has the form

$$\det \hat{P}^+\psi_1 = \varkappa\psi_2, \qquad \det \hat{B}\psi_2 = \varkappa\psi_1. \tag{12}$$

The corresponding invariant of the group with reflections $\bar{P}(SU_6)$ is equal to the product of determinants

$$\det \hat{P}^+ \det \hat{B} = \det \hat{P}^+\hat{B} = \varkappa^2. \tag{13}$$

The determinant of the matrix $\det \hat{B}\hat{P}^+$ therefore characterizes the structure of momentum space in the group $\bar{P}(SU_6)$.

Formula (13) may be regarded as the definition of the scalar product (P, P) of the vector P with itself in $\bar{P}(SU_6)$. For two vectors P and Q, the scalar product (P, Q) will be

$$(P, Q) = \frac{1}{N}\{\det(\hat{B}^P + \hat{B}^Q)(\hat{P} + \hat{Q}) - \det \hat{B}^P\hat{P} - \det \hat{B}^Q\hat{Q}\}, \tag{14}$$

where $N = 2^{12} - 2$. In $P(SU_6)$, the scalar product of the vectors P and Q is defined, respectively, by the formula

$$(P, Q) = \frac{1}{N'}\{\det(\hat{P} + \hat{Q}) - \det \hat{P} - \det \hat{Q}\}, \tag{15}$$

where $N' = 2^{11} - 2$.

The system of equations (12), in the case of the fundamental representation ψ (a 12-component quark), is equivalent to a linear equation of Dirac type, obtained by linearizing the quadratic form (9). Such an equation was proposed in papers

(^{3,6}). In this case det $\hat{B}\hat{P}^{+}$ is completely determined by the quadratic form (9). Indeed, if the Proca condition (^{3}) $(\hat{B}\hat{P}^{+})_{ij} = m^2\delta_{ij}$ is satisfied, then (13) reduces to the product of the invariants (9). The matrix condition $(\hat{B}\hat{P}^{+})_{ij} = m^2\delta_{ij}$ is equivalent to two relations, one of which is (9) and characterizes the representation $\bar{P}(SU_6)$, while the other has the meaning of an additional condition:

$$\{D_{a0c}(B_0P_a^{+} + P_0^{+}B_a) + (D_{abc}P_a^{+}B_b + iF_{abc}P_a^{+}B_b)\}A_c\psi = 0. \tag{16}$$

But in the representation (6.0) of the group $\bar{L}(SU_6)$, the generators have the form $M_a = A_a$, $N_a = A_a$. Therefore, in the representation ψ, the additional condition (16), with the aid of (8), can be written in the form

$$\zeta\psi \equiv (w_0^{+}P_0^{-} - w_n^{+}P_n^{-})\psi = 0. \tag{17}$$

Since w_α^{+} transforms like P_α^{+}, the operator on the left-hand side of (17) is an invariant.

The quantity det $\hat{B}\hat{P}^{+}$ does not depend on the order of the factors $\hat{B}$ and $\hat{P}^{+}$; therefore, it must be that $\eta = F_{abc}B_{bP}c^{+} = 0$. Although this condition has no invariant-

of this form, but together with (17) it is invariant in the sense that under the transformations $\psi \to U\psi$, $U \in L(SU_6)$, it is preserved for functions ψ satisfying (17), and $\eta\psi = 0$.

Analogously to (8), let us form the operator

$$W_\nu = M_aP_0D_{a\nu 0} + D_{a\nu m}\widetilde{P}_mM_a + iF_{\nu am}P_mN_a. \tag{18}$$

The operator W_ν commutes with $\widetilde{P}_a$ and P_a, if the condition $\eta = 0$ is fulfilled.

The characteristic features of the symmetry of elementary particles in the group $P(SU_6)$ are determined by the structure of the determinant (6) together with the definition of the scalar product (15). The role of the invariant spin is here played by the invariant (w, w) for the vector (8).

In the group with reflections, equation (12) for an arbitrary representation must also be supplemented by an invariant condition connected with the "invariant spin" (w, w), calculated by formula (14).

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Note: Figure translations are in progress. See original paper for figures.

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