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Abstract

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N. S. RYTOVA

THE COULOMB INTERACTION OF ELECTRONS IN A THIN FILM

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Real semiconductor films, as a rule, are heavily doped. The interaction of current carriers in such semiconductors often plays an important role. It is therefore of interest to clarify the influence of this interaction on the formation of the spectrum of free current carriers in a film.

By a film we shall mean a plane layer of a medium of thickness L with dielectric permittivity ε. We direct the z-axis perpendicular to the plane of the film. The electron gas is distributed in such a layer with mean density N per unit area. With the same density there is uniformly distributed the positive background charge, so that the system as a whole is neutral. It is assumed that an electron in the film is in a one-dimensional potential well with infinitely high walls. Of course, such a model of a film is an idealization, abstracting both from defects of the crystal structure and from the physical properties of the surface of a real film. However, the features due to the geometry of the problem itself are reflected by this model.

Throughout, dimensionless lengths, energies, and times are used, introduced with the aid of the scales $l_0 = L$, $E_0 = \hbar^2/2mL^2$, $t_0 = 2mL^2/\hbar$.

The wave function and spectrum of an ideal electron gas in the film have the form

$$\Psi_{\mathbf{k},n}(\mathbf{r}) = \frac{\sqrt{2}}{2\pi} e^{i\mathbf{k}\rho} \sin \pi n z, \qquad E(\mathbf{k}, n) = \pi^2 n^2 + k^2, \tag{1}$$

where ρ is the radius vector of the electron in the plane of the film; $\mathbf{k}$ is a two-dimensional wave vector describing the free motion of the electrons in the plane of the film; n is an integer quantizing the motion of the electrons in the direction of the z-axis. The main distinction between a film and an infinite crystal—its small linear size in one of the directions—leads to the fact that the regions of allowed electron states in k-space turn out to be plane sections of the Brillouin zone. Therefore, in order to elucidate film features in pure form, it is of interest to consider the limiting case of a very thin film, when effectively only the first quantum level with $n = 1$ is occupied and the electron gas becomes two-dimensional.

The fulfillment of the inequalities

$$k_0^2 \ll \theta < 3\pi^2, \tag{2a}$$

where $k_0^2 = 2\pi N L^2$, and θ is the dimensionless temperature in energy units, ensures a two-dimensional Boltzmann distribution. The distribution function of the electrons in the k-plane and the Fermi energy take, respectively, the form

$$n(k) = \frac{k_0^2}{\theta} e^{-k^2/\theta}, \qquad \mu_0 = \pi^2 + \theta \ln \frac{k_0^2}{\theta}. \tag{2b}$$

The inequalities

$$\theta \ll k_0^2 < 3\pi^2 \tag{3a}$$

lead to "two-dimensional degeneracy." In this case

$$n(k) = 0(k_0 - k), \qquad \mu_0 = \pi^2 + k_0^2. \tag{3b}$$

It is the latter case that is considered in the present work.

The energy of the electrostatic interaction of two point charges in the film is obtained by solving Poisson' s equation with the corresponding boundary conditions and has the form

$$u(\mathbf{r}_1, \mathbf{r}_2) = \int \frac{dk}{(2\pi)^2} e^{i\mathbf{k}(\rho_1 - \rho_2)} u(k; z_1, z_2),$$

where

$$u(k; z_1, z_2) = \frac{\pi S}{k} \left\{ e^{-k|z_1 - z_2|} + \frac{2\delta}{e^{2k} - \delta^2} e^k \left[\operatorname{ch} k(z_1 + z_2 - 1) + \delta \operatorname{ch} k(z_1 - z_2) \right] \right\}, \tag{4}$$

$$\delta = \frac{\varepsilon - 1}{\varepsilon + 1}, \qquad S = \frac{4L}{a_0} = \frac{4L}{\varepsilon \hbar^2 / m e^2}.$$

The problem was solved by perturbation theory for the mass operator of the causal Green' s function at $T = 0$. The first approximation in the coupling constant contains two diagrams, one of which (with an electron loop) is compensated by the term in the Hamiltonian describing the interaction of the electrons with the positive background charge. The contribution of the second diagram is

$$M(k;n,m) = -\int \frac{dk_1}{(2\pi)^2}\, n(k_1)\, u_{nm}(|\mathbf{k}-\mathbf{k}_1|), \tag{5}$$

where

$$u_{nm}(k) = 4\int_0^1\int_0^1 dz_1 dz_2\, u(k;z_1,z_2)\sin\pi z_1 \sin\pi z_2 \sin\pi n z_1 \sin\pi m z_2. \tag{6}$$

If the inequality

$$k_0 \ll 1, \tag{7}$$

is satisfied, i.e., if the film thickness becomes smaller than the mean distance between electrons, then for $k \sim k_0$ the function $u_{nm}(|\mathbf{k}-\mathbf{k}_1|)$ under the integral sign in (5) can be expanded in a series in its argument. In this case the first diagonal element u_{11} turns out to be parametrically the largest, and thus the nonideal system also becomes two-dimensional. In the approximation (7) the formula holds:

$$u_{nm}(k) = \delta_{n1}\delta_{m1}u(k) = \delta_{n1}\delta_{m1}\frac{2\pi S}{k(k+2/\varepsilon)}. \tag{8}$$

The quantity $u(k)$ plays the role of the Fourier transform of the two-dimensional interaction energy.

It can be shown that, as in the three-dimensional case, diagrams of higher approximations that contain electron loops in the interaction line diverge. Consequently, it is necessary to "dress" the interaction line, i.e., to take screening into account. The polarization operator at $E=0$ is equal to

$$\mathcal{P}(k,z_1,z_2;E=0) = 4\sum_n \pi(k,n)\sin\pi z_1 \sin\pi z_2 \sin\pi n z_1 \sin\pi n z_2. \tag{9}$$

The role of the two-dimensional polarization operator is played by the quantity $\pi(k) \equiv \pi(k,1)$:

$$\pi(k) = \begin{cases} -\dfrac{1}{2\pi}, & k < 2k_0, \\ -\dfrac{1}{2\pi}\left(1+\sqrt{1-(2k_0/k)^2}\right), & k > 2k_0. \end{cases} \tag{10}$$

The Fourier transform of the screened interaction energy for $k < 2k_0$ takes the form:

$$\tilde{u}(k) = \frac{u(k)}{1 - u(k)\pi(k)} = \frac{2\pi S}{S + k(k + 2/\varepsilon)}. \tag{11}$$

For films of ~ 100 Å, the quantity S is of order 1-10. Therefore, in the approximation adopted, the quantity $\tilde{u}(k)$ under the integral sign in (5) may be regarded as a constant:

$$\tilde{u}(k) \approx 2\pi. \tag{12}$$

Substituting (12) into (5), we obtain

$$M(k) = -{}^1/_2 k_0^2. \tag{13}$$

Result (13) means that the Coulomb interaction leads to a shift of all energies downward by a constant amount, without changing the shape of the spectrum.

The chemical potential of the system with interaction proves to be equal to

$$\mu = \mu_0 + M(k_0) = \pi^2 + {}^1/_2 k_0^2. \tag{14}$$

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Moscow State University
named after M. V. Lomonosov

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Note: Figure translations are in progress. See original paper for figures.