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**Abstract**

**Full Text**

**Physics**

**E. G. Lariontsev**

## **On the Stabilization of Certain Instabilities in the Motion of Plasma Across a Magnetic Field**

*(Presented by Academician M. A. Leontovich, 11 XII 1964)*

Some problems concerning the stability of the boundary of a plasma with a magnetic field, previously investigated for the case of a plasma at rest <sup>(1,2)</sup>, are considered for plasma flows. It is shown that the boundary of a plasma moving with a velocity that varies with depth is more stable than the boundary of a plasma at rest. The stabilization of the fluting instability is considered in the case of plane flows, and the instability with respect to necking is considered in the case of cylindrical jets.

1. We shall consider the fluting instability using the following model. Let an ideally conducting plasma occupying the region  $z > 0$  flow with velocity  $v_x(z) = U_0 + Uz/a$ , where  $a$ ,  $U_0$ , and  $U$  are constants. A gravitational force  $\rho \mathbf{g}$  acts on the plasma along the  $z$ -axis. At  $z = 0$  the plasma density changes discontinuously to zero. We denote the unperturbed magnetic field inside the plasma by  $\mathbf{B} = \{B_x, B_y, 0\}$ , and outside by  $\mathbf{H} = \{H_x, H_y, 0\}$ .

Consider perturbations

$$f(\mathbf{r}, t) = f(z) \exp i(kx + my - \omega t) \quad (1,1)$$

with wave vector  $\mathbf{k}$  perpendicular to  $\mathbf{B}$  ( $\mathbf{kB} = kB_x + mB_y = 0$ ). Such perturbations do not distort the magnetic field inside the plasma. The dispersion equation, generalizing the known result for fluting instability in a plasma at rest <sup>(1)</sup>, has the form

$$\Omega^2 + \frac{k}{\sqrt{k^2 + m^2}} \frac{du_x}{dz} \Omega = \frac{(\mathbf{kH})^2}{4\pi\rho} - \sqrt{k^2 + m^2} g, \quad (1,2)$$

where  $\Omega = kU_0 - \omega$ ,  $du_x/dz = U/a$ .

It follows from equation (1,2) that the inhomogeneity of the plasma flow velocity ( $U \neq 0$ ) has a stabilizing effect on the fluting instability. The stability condition can be represented in the form

$$(k^2 + m^2) \frac{H^2}{4\pi\rho} \sin^2(\mathbf{B}, \mathbf{H}) + \frac{1}{4} \frac{U^2 B_y^2}{a^2 B^2} \geq g\sqrt{k^2 + m^2}. \quad (1,3)$$

It is evident from this that, if the magnetic fields inside and outside the plasma are parallel, then the stabilization condition is determined only by the second term, which depends on the gradient of the velocity  $v_x(z)$ . This term is maximal when the plasma moves across the magnetic field ( $B_x = 0$ ).

2. Let us consider the analogous effect of stabilization of the instability with respect to necking in the case of a cylindrical jet with longitudinal velocity  $v_z^0(r)$ . Let a surface current parallel to the axis flow in a jet of an ideally conducting fluid. The magnetic field is absent inside the jet ( $r < a$ ), while outside it has the component  $H_\varphi = H_0 a/r$ .

In the linear approximation, for perturbations of the necking type

$$f(\mathbf{r}, t) = f(r) \exp i(kz - \omega t) \quad (2,1)$$

the system of hydrodynamic equations for  $r < a$  reduces to one equa-

$$\left[ \frac{\Omega^2(r)}{r} f' \right]' - \frac{k^2 \Omega^2(r)}{r} f = 0, \quad (2,2)$$

where  $f = rv_r/\Omega(r)$ ,  $\Omega(r) = \omega - kv_z^0(r)$ .

Using the boundary conditions on the perturbed surface of the cylinder  $r = a + \xi$ ,

$$v_r(a) = i\Omega\xi,$$

$$p_0 + p_1 = \frac{H_\varphi^2}{8\pi}(a + \xi) = \frac{H_0^2}{8\pi} - \frac{H_0^2}{4\pi} \frac{\xi}{a}$$

and the relation

$$-\frac{\Omega(r)}{r} \frac{d}{dr}(rv_r) + v_r k \frac{dv_z^0}{dr} = -i \frac{k^2 p_1}{\rho}, \quad (2,3)$$

we obtain the dispersion equation in the form

$$\alpha\Omega^2 - \beta\Omega + k^2 v_H^2 = 0, \quad (2,4)$$

where  $\alpha = (rv_r)' / v_r|_{r=a}$ ,  $\beta = ka dv_z^0 / dr|_{r=a}$ ,  $v_H^2 = H_0^2 / 4\pi\rho$ ,  $\Omega = \Omega(a)$ .

For a jet with a parabolic velocity profile  $v_z^0(r) = U_0 + Ur^2/a^2$ , equation (2.2) reduces to the Bessel equation

$$v_r'' + rv_r' - (k^2 + 1/r^2)v_r = 0,$$

whence it follows that

$$\alpha = kaI_0(ka)/I_1(ka). \quad (2.5)$$

Since  $\alpha$  does not depend on  $\Omega$ , equation (2.4) is quadratic and has the roots

$$\Omega_{1,2} = \frac{\beta}{2\alpha} \pm \frac{\sqrt{\beta^2/4 - \alpha k^2 v_H^2}}{\alpha}. \quad (2.6)$$

It follows from (2.6) that a thin current-carrying cylindrical jet is more stable with respect to constrictions than the corresponding stationary cylinder ( $\beta = 0$ ). The stability condition for a jet with surface current has the form

$$\beta^2/4k^2 = U^2 \geq \alpha v_H^2. \quad (2.7)$$

Long-wavelength perturbations ( $ka \ll 1$ ) are most easily stabilized, since

$$\alpha \simeq 2 \quad \text{for } ka \ll 1,$$

$$\alpha \simeq ka \quad \text{for } ka \gg 1. \quad (2.8)$$

3. Analogous results are obtained in the study of the stability of a hydrodynamic jet with surface tension. Formulas (2.2)–(2.8) can be used in this case as well; it is only necessary to replace  $v_H^2$  by the quantity

$$v_T^2 = \frac{T}{\rho a}(1 - k^2 a^2),$$

where  $T$  is the coefficient of surface tension.

The stability condition for a jet with a parabolic velocity profile with respect to long-wavelength constrictions, in accordance with (2.7), (2.8), has the form

$$U^2 > \frac{2T}{\rho a}.$$

4. In the experimental investigation of accelerating plasma jets with their own currents <sup>(3)</sup>, a high stability of current filaments in the jets with respect to pinches was found. It is possible that the stabilization effect of pinches due to the nonuniformity of the jet velocity, considered above, plays an important role here.

In <sup>(4)</sup> an experiment is cited showing that a poorly conducting current-carrying jet proves to be more stable with respect to pinches than the corresponding plasma cylinder at rest.

There is an analogous example in hydrodynamics. It is known that a liquid cylinder at rest is unstable with respect to pinches under the action of surface-tension forces. But a jet (for example, a water jet flowing from a tap) may prove stable. Apparently, the stabilization effect associated with the nonuniformity of the jet velocity is manifested here as well.

In <sup>(4)</sup> the stability of a poorly conducting current-carrying jet with surface tension was considered. A uniform longitudinal current flowed in the jet and the velocity profile was parabolic. As was noted in <sup>(4)</sup>, the results obtained do not agree with the experimental data.

In the special case of a hydrodynamic jet (with zero current), the results of <sup>(4)</sup> should reduce to those obtained by us in Section 3. However, in our notation the dispersion equation of <sup>(4)</sup> for a hydrodynamic jet has the form

$$\alpha\Omega^2 + k^2v_T^2 = 0. \quad (4.1)$$

Equation (4,1) is the dispersion equation for a cylinder at rest with surface tension, and it follows from it that the cylinder is unstable with respect to long-wavelength pinches. The term  $\beta\Omega$ , responsible for the stabilization of pinches, is absent in the dispersion equation of <sup>(4)</sup>. This is connected with the fact that an error was made in <sup>(4)</sup>. An incorrect relation was obtained connecting the perturbations of pressure and velocity. It differs from equation (2,3) of the present work in that the term  $v_r k dv_z^0/dr$  was not taken into account.

Moscow State University  
named after M. V. Lomonosov

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*Note: Figure translations are in progress. See original paper for figures.*

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