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Abstract

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ON A HYDRODYNAMIC THERMAL EXPLOSION

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As is known, in a system in which an exothermic chemical reaction takes place, conditions are possible under which a progressive rise in temperature occurs, leading to the so-called thermal explosion [1]. In the present work it is shown that a phenomenon analogous to a thermal explosion can occur in the flow of a chemically inert viscous liquid.

Let us consider the simplest example of a steady axisymmetric laminar flow of a viscous incompressible liquid in an infinitely long circular tube of radius r_0. The flow occurs under the action of a constant pressure gradient; the density of the liquid is taken to be constant. Then the system of equations of motion and heat conduction, with allowance for energy dissipation, will have the form

$$\frac{\partial}{\partial r}\left(\mu\frac{\partial v}{\partial r}\right)+\frac{\mu}{r}\frac{\partial v}{\partial r}-\frac{\partial P}{\partial z}=0, \qquad \frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\mu}{\lambda J}\left(\frac{\partial v}{\partial r}\right)^2=0. \tag{1}$$

Boundary conditions:

$$\text{at } r=r_0 \quad v=0,\ T=T_0; \qquad \text{at } r=0 \quad \partial v/\partial r=0,\ \partial T/\partial r=0. \tag{2}$$

Notation: v is the velocity; $-\partial P/\partial z=b$ is the pressure gradient; μ is the dynamic coefficient of viscosity; T is the temperature; T_0 is the temperature of the tube walls; λ is the coefficient of thermal conductivity of the liquid; J is the mechanical equivalent of heat.

In this problem, it is of interest to consider liquids with a strong dependence of viscosity on temperature. For some liquids, this dependence over a sufficiently wide temperature range is satisfactorily described by an exponential formula of the form [2]

$$\mu=\mu_0 e^{U/RT}, \tag{3}$$

where μ_0 and U are constants; R is the gas constant; T is the absolute temperature.

We reduce the system of equations (1), taking (3) into account, to dimensionless form

$$\frac{1}{x}\frac{\partial}{\partial x}\left(e^{-\theta/(1+\beta\theta)}x\frac{\partial w}{\partial x}\right)+1=0, \qquad \frac{\partial^2\theta}{\partial x^2}+\frac{1}{x}\frac{\partial\theta}{\partial x}+16\chi e^{-\theta/(1+\beta\theta)}\left(\frac{\partial w}{\partial x}\right)^2=0, \tag{4}$$

where the following notation has been introduced for dimensionless variables and parameters: dimensionless variables:

$$w=\frac{\mu_0}{br_0^2}\exp\left(\frac{U}{RT_0}\right)v, \qquad \theta=\frac{U}{RT_0^2}(T-T_0), \qquad x=\frac{r}{r_0};$$

dimensionless parameters:

$$\chi=\frac{Ub^2r_0^4}{16J\mu_0\lambda RT_0^2}\exp\left(-\frac{U}{RT_0}\right), \qquad \beta=\frac{RT_0}{U}$$

(in the case of a strong dependence of viscosity on temperature, $\beta \ll 1$).

The boundary conditions (2) take the form

$$\text{at } x=1 \quad w=0,\ \theta=0; \qquad \text{at } x=0 \quad \partial w/\partial x=0,\ \partial\theta/\partial x=0. \tag{5}$$

The system of equations (4) can be reduced to a single equation as follows. Integrating once the first equation of system (4), taking into account the second boundary condition (5), we obtain

$$xe^{-\theta/(1+\beta\theta)}\,dw/dx=-1/2x^2. \tag{6}$$

Eliminating the velocity gradient from (6) and the second equation of system (4), we have

$$\frac{d^2\theta}{dx^2}+\frac{1}{x}\frac{d\theta}{dx}+4\chi x^2e^{\theta/(1+\beta\theta)}=0.$$

By the change of variable $x^2=\xi$, this equation is brought to the form

$$\frac{d^2\theta}{d\xi^2}+\frac{1}{\xi}\frac{d\theta}{d\xi}+\chi e^{\theta/(1+\beta\theta)}=0. \tag{7}$$

To determine the temperature field it is necessary to integrate equation (7) under the boundary conditions

$$\text{at } \xi = 1 \quad \theta = 0; \qquad \text{at } \xi = 0 \quad d\theta/d\xi = 0. \tag{8}$$

It is interesting to note that the equation obtained coincides exactly with the equation of the stationary theory of thermal explosion (see, for example, ([3],[4])). Thus, many conclusions of this theory can be carried over to the case under consideration. First of all, it follows from this theory that equation (7), with boundary conditions (8), has a solution only for $\chi < \chi_{\text{cr}}$, where χ_{cr} is a certain critical value. For $\chi > \chi_{\text{cr}}$, equation (7) has no solution, and a stationary distribution of temperature and velocities is impossible. In this regime, the heat released by internal friction does not have time to be removed through the tube walls and leads to a progressive increase in temperature (a hydrodynamic thermal "explosion"). According to D. A. Frank-Kamenetskii ([5]), for small values of β*

$$\chi_{\text{cr}} = 2.00, \qquad \theta_{\text{cr}} = 1.39$$

(θ_{cr} is the greatest stationary heating of the liquid on the tube axis).

Let us now consider the stationary distribution of temperatures and velocities for $\beta \to 0$. Having found θ as a function of x, from (6) one can determine the velocity profile. The solution of equation (7) for $\beta = 0$, after satisfying the second boundary condition (8), can be written in the form ([6])

$$\theta = \ln \frac{8}{\chi} - 2\ln(a\xi^2 + 1/a), \tag{9}$$

where a is a constant of integration. Satisfying the first boundary condition (8), to determine the constant a we obtain a quadratic equation with roots

$$a_1 = \sqrt{2/\chi} + \sqrt{2/\chi - 1}, \qquad a_2 = \sqrt{2/\chi} - \sqrt{2/\chi - 1}.$$

These two roots correspond to two different temperature profiles. As is known from the theory of thermal explosion, of the two solutions only one is stable (in the present case the stable solution corresponds to the root a_2, which in what follows we shall denote by a).

Replacing ξ in (9) by x and substituting in (6), one can write

$$\frac{dw}{dx} = -\frac{4}{\chi a^2} \frac{x}{(x^4 + 1/a^2)^2}.$$

Integrating this with allowance for (5), we obtain

$$w = \frac{a}{\chi}\left(\frac{a}{1+a^2} + \operatorname{arc\,tg} a - \frac{ax^2}{1+a^2x^4} - \operatorname{arc\,tg} ax^2\right). \tag{10}$$

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* In this case, in equation (7) one may put $\beta = 0$.

Hence the dimensional expression for the velocity on the tube axis has the form

$$v_0 = \frac{br_0^2}{\mu(T_0)} \cdot \frac{a}{\varkappa}\left(\frac{a}{1+a^2} + \operatorname{arc\,tg} a\right). \qquad (11)$$

The volume flow rate of the liquid is expressed by the formula

$$Q = \pi b r_0^4 a^2 / \varkappa \mu(T_0)(1+a^2). \qquad (12)$$

If one takes into account that, for small values of $\varkappa$, $a \to \sqrt{2\varkappa}/4$, then as $\varkappa \to 0$, which corresponds to isothermal flow of the liquid, we obtain the formulas

$$Q_{\mathrm{iz}} = \pi b r_0^4 / 8\mu(T_0),$$

$$v_{0\mathrm{iz}} = b r_0^2 / 4\mu(T_0), \qquad (13)$$

which coincide with the known Poiseuille formulas (^{7}).

Comparing (11), (12) with (13), we find

$$Q = \frac{8a^2}{\varkappa(1+a^2)} Q_{\mathrm{iz}},$$

$$v_0 = \frac{4a}{\varkappa}\left(\frac{a}{1+a^2} + \operatorname{arc\,tg} a\right) v_{0\mathrm{iz}}$$

or, under critical conditions,

$$Q_{\mathrm{cr}} = 2Q_{\mathrm{iz}}, \qquad v_{0\mathrm{cr}} = (1+\pi/2) v_{0\mathrm{iz}}.$$

Fig. 1. Intensity of heat sources at $\varkappa = \varkappa_{\mathrm{cr}}$: **1**—mechanical, **2**—chemical

Taking as the characteristic velocity the mean velocity of liquid flow, the expressions for the Reynolds number and the critical value of the parameter $\varkappa$ may be written in the form

$$\mathrm{Re} = \frac{r_0^3 \rho b}{4\mu^2(T_0)}, \qquad \varkappa_{\mathrm{cr}} = \frac{U}{RT_0^2} \frac{(\mathrm{Re})^2 \mu^3(T_0)}{r_0^2 \rho^2 \lambda J}. \qquad (14)$$

These formulas make it possible, at values of Re ensuring laminarity of the flow, to calculate the critical conditions of a hydrodynamic thermal "explosion."

Fig. 2. Temperature profile at $\chi = \chi_{\text{cr}}$: 1—in a hydrodynamic thermal "explosion," 2—in a chemical thermal "explosion"

Figure 1: Fig. 2. Temperature profile at $\chi = \chi_{\text{cr}}$: 1—in a hydrodynamic thermal "explosion," 2—in a chemical thermal "explosion"

Table 1

T_0, °K	r_0, cm	b, atm/cm	v_0, m/sec	Q, cm^3/sec	ΔT_{cr}, deg
40	1.79	$2.95 \cdot 10^{-3}$	18.7	7351	21.8
50	0.69	$1.52 \cdot 10^{-2}$	26.3	1523	23.2
60	0.28	$7.17 \cdot 10^{-2}$	36.3	343	24.6

In Table 1, as an example, critical values of the principal quantities characterizing the flow are given for glycerin at Re = 500. In the calculation the following values of the constants were used: $\mu_0 = 6.8 \cdot 10^{-9}$ poise, $U = 12\,520$ cal/mole*, $\lambda = 6.67 \cdot 10^{-4}$ cal/cm • sec • deg, $\rho = 1.26$ g/cm^3, $J = 4.18 \cdot 10^7$ erg/cal. In the present example $\beta \approx 0.05 \ll 1$, which makes it possible, with satisfactory accuracy ($\sim 5\%$), to use the value $\varkappa_{\text{cr}} = 2$ for calculating the critical conditions.

Let us note certain differences between thermal "explosions" of chemical and hydrodynamic origin.

* Processing of the data given in (^{8}) showed that the formula $\mu(T) = 6.8 \cdot 10^{-9} \exp(12\,520/RT)$ poise describes the experimental values of μ, with good accuracy (2–3%), in the temperature range 40–100°C.

1. Heat release during the flow of a viscous liquid ultimately corresponds to a zero-order "reaction." The so-called "burnout" in a hydrodynamic "explosion" is absent, and no analogue of the well-known criterion γ exists (see, for example, (4)). In this connection, the stationary theory of the hydrodynamic "explosion" is rigorous, and not an approximation, as in the case of a chemical explosion.

Fig. 2. Temperature profile at $\chi = \chi_{\text{cr}}$: 1—in a hydrodynamic thermal "explosion," 2—in a chemical thermal "explosion."

2. The greatest intensity of chemical heat sources is located at the center of the system, whereas that of mechanical sources is near the surface (Fig. 1). As a consequence of this, the stationary temperature profile in the hydrodynamic problem is flatter in the central layers and steeper in the surface layers (Fig. 2).

Thus, the results obtained in this work indicate that, in the case of a strong (nonlinear) dependence of viscosity on temperature, owing to energy dissipation there may exist critical conditions for the thermal regime of liquid flow. We note that the existence of critical conditions is characteristic of many thermal problems with nonlinear heat sources (thermal breakdown of dielectrics, thermal explosion, etc.).

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Note: Figure translations are in progress. See original paper for figures.

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