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Abstract

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DIELECTRIC ELECTRONICS AND THE QUADRATIC LAW OF CURRENTS LIMITED BY SPACE CHARGE*

PHYSICS

1. In recent years a new branch of the physics of solids and its applications has arisen and is developing intensively, which may be called **dielectric electronics**. Its basis is formed by the processes of passage of emission currents in dielectrics. Like emission currents in a vacuum, like injection currents through a *p*—*n* junction in semiconductors, these processes can be used to create elements and units of electronic engineering and radio electronics.

Without dwelling on a comparative analysis of the advantages and disadvantages of vacuum, semiconductor, and dielectric electronics, let us note the inseparable connection of the latter with film devices and, consequently, its microminiature character by its very physical nature. The nonequilibrium conductivity of a nonmetallic crystal, caused by emission from an electrode, rapidly decreases with distance from the contact and already at small distances becomes very small. Therefore only in films can it dominate throughout the entire volume over the conductivity of the material itself. As has already been noted ([5]), a nonmetallic solid can be regarded as a dielectric only at a sufficiently small thickness (a film!), since otherwise in the depth of the layer the concentration of equilibrium charge carriers will be greater than the concentration of carriers that have entered from the metal.

2. The problem of currents in a dielectric layer between two arbitrary metallic contacts is determined by the system of equations

$$j = \text{const}; \qquad dE/dx = -n; \qquad dn/dx + nE = j. \tag{1}$$

The units of concentration, distance, current density, potential, and field strength are

$$\bar{n} = n_1^*; \qquad \bar{x} = x_{\text{Deb}} = \sqrt{kT/4\pi q^2 n_1^*}; \qquad \bar{j} = qDn_1^*/x_{\text{Deb}};$$

$$\bar{\psi} = kT/q; \qquad \bar{E} = kT/qx_{\text{Deb}}, \tag{2}$$

where n_1^* is the boundary concentration at the cathode (at $x = 0$).

For currents small in comparison with the Richardson current, the boundary conditions reduce to prescribing the concentrations at the contacts

$$n(0) = 1; \qquad n(L) = n_2. \tag{3}$$

The exact solution of this problem leads to complicated expressions, representable in finite form only in special Airy or Bessel functions ([1–9]) and containing integration constants whose determination from the boundary conditions generally requires the use of numerical methods of calculation and machine mathematics. For this reason the overwhelming majority of works on the investigation of injection currents in dielectrics is based on an approximate solution obtained by Mott and Gurney ([10]) under the assumption that the diffusion component of the current is small in comparison with the drift component: $dn/dx \ll nE$. Neglecting dn/dx in the sys-

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system (1), Mott and Gurney obtain

$$E = -\sqrt{2}(-C - jx)^{1/2}; \qquad n = -\frac{j}{2}(-C - jx)^{-1/2};$$

$$\psi = -\frac{2\sqrt{2}}{3j}(-C - jx)^{3/2}, \tag{4}$$

whence there follows the following relation between the current j and the applied voltage V $(x_1 = -c/j)$:

$$V = \sqrt{8|j|/9}\left[(L + x_1)^{3/2} - x_1^{3/2}\right]. \tag{5}$$

The characteristic length

$$x_1 = |j|/2 \tag{6}$$

is determined from the boundary condition $n = 1$ at the cathode. For $x_1 \ll L$, (5) gives the quadratic law for currents limited by space charge (SCLC) in dielectrics,

$$j = -\frac{9}{8}V^2/L^3. \tag{7}$$

The question of the limits of applicability of the quadratic law (7) has remained unresolved up to now. Neglect of the diffusion current lowers the order of the equation for E, as a result of which solution (4) contains only one integration constant and does not take into account the influence of the second contact (the anode) on the SCLC regime in dielectrics. Many authors regard the quadratic Mott-Gurney law as just as fundamental for SCLC in dielectrics as Langmuir' s 3/2 law is for vacuum (see, for example, ([11])). According to estimates by Mott and Gurney, the law $j \propto V^2$ is valid for $V \gg kT/q$, i.e., practically at all real voltages up to such large values that the drift of injected charge carriers produces a uniform distribution of them in dielectrics (the region of Ohm' s law).

Consideration of the Mott-Gurney law on the basis of an exact solution of the problem leads to a significantly more stringent condition ([5])*

$$j \gg 10 \quad \text{or} \quad V^* \gg \frac{3kT}{q}\left(\frac{L^*}{x_{\text{Deb}}}\right)^{3/2}. \tag{8}$$

It follows from this formula that, for sufficiently small values of the work function from the cathode into the dielectric, the region of applicability of the law $j \propto V^2$ shifts toward higher voltages and becomes very limited. Numerical calculations carried out for some particular cases, as well as experimental data, show, however, that the quadratic SCLC dependence in dielectrics is observed quite often and over sufficiently wide ranges ([1,4,6,12,13]), etc.

The purpose of the present work is a general analytical estimate of the region of applicability of the quadratic SCLC law in dielectrics.

3. The general expression for the current-voltage characteristic of a dielectric layer situated between two metallic contacts was obtained in ([5]):

$$V = 2\ln\left[u_0^{1/3} Z_{1/3}(u_0)/\gamma u_L^{1/3} Z_{1/3}(u_L)\right]. \tag{9}$$

Here

$$u(x) = \frac{\sqrt{2}}{3|j|}(C + jx)^{3/2}.$$

It was also shown in ([5]) that the Mott-Gurney solution corresponds to neglecting diffusion in comparison with drift throughout the entire dielectric, except for the near-anode region, which in this regime becomes so thin that its space charge does not affect the distribution of the field and potential in the dielectric,

* All designations of physical quantities and functions are the same as in article ([5]); in particular, asterisks de

and the diffusion processes occurring in it lead to the appearance of an additional term in the expression for the concentration, one that is significant only near the anode and ensures fulfillment of the boundary conditions at the anode. Noting from the writing of the quadratic law in dimensional units

$$j^* = -\frac{9\chi\mu}{32\pi}\frac{V^{*2}}{L^{*3}},$$

that the boundary concentration of charge carriers at the cathode does not enter into it, it is natural to expect that the near-cathode region can similarly be separated out. The assumption that the inequality

$$w(x) = \frac{\sqrt{2}}{3|j|}(-C - jx)^{3/2} \gg 1, \tag{10}$$

which leads to approximation of the exact solution by the Mott–Gurney formula, is satisfied not everywhere, but only outside the near-cathode region, can substantially soften the estimate (8).

Since condition (10) must certainly be fulfilled at the anode, formula (9) is simplified with the aid of asymptotic expressions for $Z_{1/3}(w)$ ([5], [14]). For $w_L \gg 5$

$$V = 2w_L + 2\ln\left|\frac{u_0^{1/3} Z_{1/3}(u_0) w_L^{1/6}\left(1 + \sqrt{1 + \frac{2}{(9|j|^2 w_L^2)^{1/3}}}\right)}{\gamma\sqrt{2\pi}}\right|. \tag{11}$$

Here the second constant of integration B', entering into $Z_{1/3}(w_L)$, is determined from the condition at the anode and turns out to be very small. When the inequalities

$$w_L \gg 1; \qquad \left|\frac{j}{C}\right| \gg \frac{1}{L}, \qquad \left|\ln\left|\frac{u_0^{1/3} Z_{1/3}(u_0) w_L^{1/6}\left(1 + \sqrt{1 + \left(\frac{2}{9|j|^2 w_L^2}\right)^{1/3}}\right)}{\gamma^2\pi}\right|\right| \ll w^L \tag{12}$$

are fulfilled, expression (11) coincides with the quadratic law (7):

$$V \simeq 2w_L \simeq \frac{2\sqrt{2}}{3}|j|^{1/2} L^{3/2}. \tag{13}$$

To determine the conditions for fulfillment of inequalities (12), it is necessary to find the functional dependence $C(j)$. This dependence can be determined from the boundary condition at the cathode. Substituting into this condition the exact solution of system (1) (formula (11) in ([5])), we arrive at the equations

$$\frac{1}{C} = 1 + \left[\frac{Z_{-2/3}(u_0)}{Z_{1/3}(u_0)}\right]^2; \qquad u_0 = \frac{\sqrt{2}}{3|j|}C^{3/2}, \tag{14}$$

which make it possible to determine C as a function of j and L. As long as w_L and $|j|L$ are sufficiently large, the functions $Z_{1/3}(u_0)$ and $Z_{-2/3}(u_0)$ are insensitive to the values of B' and $\sqrt{1+1/|j|L}$, and formula (14) determines C as a function of j only. For $L = \infty$ this universal function is valid down to $j = 0$. It is shown in Fig. 1; moreover, its positive branch was calculated from formulas (14), respectively for real u_0 and for $u_0 = -iw_0$ imaginary. For $|j|$ larger than those values which are shown in Fig. 1b, the analytic dependence ([5]) holds:

$$C \simeq -j^2/2.$$

Noting that, when condition (12) is fulfilled, the approximate equality (13) holds, one can transform inequality (12) to the following more convenient form:

$$V \gg 10; \tag{15}$$

$$|j|/|C| \gg 10/L; \tag{16}$$

$$V \gg 20\left|\ln\left|2u_0^{1/3}Z_{1/3}(u_0)\right|\right|. \tag{17}$$

In addition to condition (15), which corresponds to the Mott-Gurney criterion, the quadratic law for space-charge-limited currents in dielectrics is determined by two further inequalities—

properties (16) and (17). Noting that $|j| = 0$ corresponds to the root $u_0^0 \simeq 2.38$ of the function $Z_{1/3}(u_0)$, we are convinced that, as $u_0 \to 2.38$, the ratio $|j|/C = \sqrt{2C}/3u_0 \to 0$. Consequently, this ratio varies from 0 to ∞ in the region of the first branch of the curve $C(j)$ (Fig. 1a) and from ∞ to 0 in the region of the second branch (Fig. 1b). Therefore inequality (16) bounds twice the range of currents in the dielectric corresponding to the quadratic law—from below for $j < 1.82$ and from above for $j > 1.82$. In the second region $|C|/|j| = x_1$, and the corresponding limitation is connected with the transition of the SCLC into Ohm' s law (see (4) and (6)). For $C(j) > 0$, the value of $|j|$ corresponding to condition (16), for all real values of L, lies in the vicinity of $j = 0$.

Fig. 1

Fig. 1

Figure 1: Fig. 1

Condition (17) limits the range of applicability of the quadratic law only from below, since for $j > 1.82$ the function $u_0^{1/3} Z_{1/3}(u_0)$ has no roots and the logarithmic term in (11) is small.

The true lower boundary of the quadratic SCLC law in dielectrics is the largest of the values V determined by conditions (15), (16), and (17).

The results of calculating $C(j)$ from formulas (14) show that, for $j \lesssim 3 \cdot 10^{-3}$ ($L \gtrsim 200$), u_0 differs little from its limiting value $u_0^0 = 2.38345$. In this case, as follows from (14), the following analytic dependence is valid for C:

$$C = \sqrt[3]{(3u_0^0|j|)^2/2}. \tag{18}$$

Substituting (18) into (16) and using (7), we obtain, instead of (16),

$$V \gtrsim 150. \tag{19}$$

It can be shown that, at small currents, inequality (19) is the strongest of the three conditions given above and that, beginning with $V \simeq 4$ V, the quadratic SCLC law is satisfied for all L.

Conditions (15), (16), and (17) show that the quadratic section on the current-voltage characteristic of emission currents in dielectrics arises considerably earlier than when, throughout the entire dielectric layer, there is established a distribution of fields and concentrations corresponding to the Mott-Gurney solution (4). However, although the limiting value of $|j|$ determined by conditions (16) or (17) is very small, these conditions turn out, as a rule, to be considerably more stringent than the Mott-Gurney criterion (15). We shall confine ourselves to one example illustrating what has been said.

Let us consider CdS films of thickness $L^* = 0.1;\ 1;\ 10\ \mu$. With contacts having a work function $\varphi^* = 0.1$ eV, $n_1^* = 10^{17}$ cm^{-3}, $x_{\text{deb}} = 10^{-6}$ cm and, consequently, $L = 10;\ 10^2;\ 10^3$. The corresponding minimum values V^* for SCLC are found to be $0.78;\ 3.6;\ 3.9$ V.

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Note: Figure translations are in progress. See original paper for figures.