ON THE QUESTION OF THE EXISTENCE OF A SECOND PRESSURE PEAK IN A LUBRICATING LAYER

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Abstract

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ON THE QUESTION OF THE EXISTENCE OF A SECOND PRESSURE PEAK IN A LUBRICATING LAYER

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1°. It is known that the pressure in a lubricating layer located between deforming surfaces has at least one maximum in the interval of the region of friction. In 1946 A. I. Petrusevich advanced the hypothesis of the existence of one more, moreover sharp, maximum of pressure to the left of the already known one ("the second pressure peak"). The presence of this second maximum, which is a stress concentrator, should determine the fatigue strength of the surface. Hence it follows that the solution of the question of the existence of a second pressure peak is of great importance. However, A. I. Petrusevich' s hypothesis has up to the present time been neither mathematically nor experimentally proved or refuted. True, in a paper by D. S. Kodnir ([3]) a mathematical proof was given of the absence of a second peak, but this proof is based on the study of an approximate equation for the function $k(z)$ (the pressure function).

In the present paper a mathematical proof is proposed of the absence of a second sharp peak, based on consideration of a refined integral equation for the function $k(z)$. This integral equation ([3]) is equivalent to the system of equations

$$H(z) = 1 - a^2 + z^2 + D \int_a^b k(t) \ln \left| \frac{t-a}{t-z} \right| dt; \tag{1}$$

$$\frac{dk(z)}{dz} = \frac{1 - H(z)}{H^3(z)} e^{B_n k(z)}, \qquad a < 0 < b, \tag{2}$$

where $k(z)$, $H(z)$ are unknown functions; D, B_n are constants. Concerning the functions $k(z)$, $H(z)$, satisfying this system and certain conditions determined by the hydrodynamic regime in the lubricating layer, the following is known: 1) $k(z) > 0$ in the interval $(a; b)$ and $k(a) = k(b) = 0$, $k(z)$ has a relative maximum at the point $z = 0$; 2) $H(a) = H(0) = 1$, $H(z) > 1$ for $z > 0$; 3) the functions $k(z)$ and $H(z)$ are continuous and have continuous derivatives at least up to the second order.

Our task is to prove the absence of a sharp pressure peak in the interval $(a; 0)$, i.e., the absence in the interval $(a; 0)$ of a vertex of the graph of the function $k = k(z)$ with a small base.

2°. Suppose that, in addition to the maximum point $z = 0$, there exists another maximum point $z = z_1$, $a < z_1 < 0$, and, consequently, a minimum point $z = z_2$, $z_1 < z_2 < 0$. Denote $z_2 - z_1 = h$. We shall prove that this supposition leads to a contradiction if h is sufficiently small. Thereby it will be proved that the existence of a sharp second pressure peak is impossible.

From our supposition it follows that $H(z_1) = H(z_2) = 1$; $H(z) < 1$ if $a < z < z_1$; $H(z) > 1$ if $z_1 < z < z_2$. Hence we conclude

$$d = (dH/dz)_{z=z_2} - (dH/dz)_{z=z_1} \leq 0. \tag{3}$$

On the other hand, substituting dH/dz from (1), we obtain:

$$d = 2(z_2 - z_1) + D \int_a^b k(t) \left(\frac{1}{t - z_2} - \frac{1}{t - z_1} \right) dt. \tag{4}$$

3°. Denote:

$$M_0 = \max_{a \leq z \leq b} k(z), \quad M = k(z_1), \quad m = k(z_2) \quad (0 < m < M),$$

$$L(z) = (b-a)\ln(b-a) - (z-a)\ln(z-a) - (b-z)\ln(b-z) + (z-a)\ln 2, \quad L_0 = \max_{a \leq z \leq 0} L(z).$$

From equation (1) we obtain the estimates ($a \leq z \leq 0$):

$$H(z) \leq 1 - a^2 + z^2 + DM_0 L(z), \tag{5}$$

$$H(z) \leq 1 - a^2 + z^2 + DM_0 L_0, \tag{6}$$

$$H(z) \leq 1 + DM_0 L_0. \tag{7}$$

There exists a number a_0, $a < a_0 < 0$, such that $dk(z)/dz > 0$ in the interval $a < z < a_0$. Consider a number h_0, $0 < h_0 < \min(a_0 - a,\ b)$. Put

$$I(\alpha, h) = \left(\int_a^{\alpha - h} + \int_{\beta + h}^b \right) k(t) \left(\frac{1}{t - \beta} - \frac{1}{t - \alpha} \right) dt,$$

where $\beta = \alpha + h$, $a_0 \leq \alpha < 0$, $0 < h < h_0$. It is easy to see that $I(\alpha, h) > 0$. For sufficiently small h_0 we shall have

$$\inf_{\substack{a_0 \le \alpha < 0 \\ 0 < h < h_0}} \{I(\alpha, h)\} = K_0 > 0. \tag{8}$$

From (2), taking into account that $k(a) = 0$, we obtain:

$$k(z) = -\frac{1}{B_n} \ln(1 - \omega(z)), \qquad \omega(z) = B_n \int_a^z \frac{1 - H(t)}{H^3(t)}\, dt,$$

$0 < \omega(z) < 1$. Using estimate (7), we arrive at the inequality

$$M - m < e^{B_n M^0} D M_0 L_0 (z_2 - z_1). \tag{9}$$

4°. Let $a_0 < z_1 < 0$. Equality (4) can be written as

$$d = 2h + DI(z_1, h) + DI_1(z_1, h), \tag{10}$$

where

$$I_1(z_1, h) = \int_{z_1 - h}^{z_2 + h} k(t) \left(\frac{1}{t - z_2} - \frac{1}{t - z_1} \right) dt. \tag{11}$$

Using the representation of the function $k(t)$ by Taylor' s formula in neighborhoods of the points z_1 and z_2, one can obtain the relation

$$I_1(z_1, h) = -(M - m) \ln 2 + \frac{5}{2} E(z_1, h) h^2,$$

where

$$|E(z_1(h))| < M_0'', \qquad M_0'' = \max_{z_1 - h \le z \le z_2 + h} |k''(z)|.$$

Thus,

$$d = DI(z_1, h) + [2 - E_0(z_1, h)]h + \frac{5}{2} DE(z_1, h) h^2, \tag{12}$$

where $0 < E_0(z_1, h) < e^{B_n M^0} D^2 M_0 L_0 \ln 2$. From (12) it is clear that for sufficiently small h the right-hand side of equality (12) is positive, which contradicts (3).

5°. It is natural to include in the definition of a sharp peak the requirement that, at the summits of the peak, the curvature have an extremum. Then

$$M_0'' = \max_{z_1 - h \le z \le z_2 + h} |k''(z)| = \max(|k''(z_1)|,\ |k''(z_2)|).$$

On the other hand, from (2) we obtain

$$k''(z_1) = -\,(dH/dz)_{z=z_1}\, e^{B_{nM}}, \qquad k''(z_2) = -\,(dH/dz)_{z=z_2}\, e^{B_{nm}}. \tag{13}$$

Hence it follows easily that

$$M_0'' \le |k''(z_1)| + k''(z_2) \le -de^{B_{nM}} \le -de^{B_{nM}0}. \tag{14}$$

Therefore from (12) we obtain the inequality

$$d\left(1 - \frac{5}{2} De^{B_{nM}0} h^2\right) > DK_0 + [2 - E_0(z_1, h)]\, h. \tag{15}$$

Under the assumption $d \le 0$, inequality (15) will be contradictory for sufficiently small h.

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Note: Figure translations are in progress. See original paper for figures.