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Abstract

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PHYSICS

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DYNAMICS OF AN OPTICAL QUANTUM GENERATOR WITH TWO TYPES OF OSCILLATIONS

Already in the first works chaotic pulsations of the radiation power of optical quantum generators were noted, and in subsequent experiments a regime of regular undamped oscillations of the radiation intensity was obtained ([1,2]). However, there is still no satisfactory explanation of undamped radiation pulsations of optical quantum generators, although there are a number of works in which attempts are made to describe such pulsations within the framework of rate equations on the basis of various assumptions about the mechanism of losses in the resonator ([3]). The dependence of the regimes in an optical quantum generator on the properties of the resonator ([4]) indicates that the explanation of these regimes should be sought within the framework of a multimode model. We have carried out an investigation of the simplest multimode model—an optical quantum generator with two types of oscillations.

The main difficulty in solving the problem lies in describing the polarization of the substance under the action of two types of oscillations. Although the initial equations may be the system (1), previously used to describe a single-mode generator ([5,6]), a nontrivial point is the extraction from (1b) of the polarization components that excite the corresponding types of oscillations in the resonator:

$$\ddot{E} + \frac{\omega_1}{Q}\dot{E} + \omega_1^2 E = -4\pi\ddot{P}; \tag{1a}$$

$$\ddot{P} + \frac{2}{T_2}\dot{P} + \omega_\ell^2 P = -\frac{2d_{12}\omega_{12}}{\hbar}ER; \tag{1b}$$

$$\dot{R} + \frac{1}{T_1}(R - R_0) = \frac{2}{\hbar\omega_{12}}\left(\dot{P} + \frac{1}{T_2}P\right)E. \tag{1c}$$

Here E is the field; P is the polarization of the substance; ω_ℓ is the frequency of the emission line of the substance; ω_1 is the natural frequency of the resonator; Q is its quality factor; R is the density of the inverted population; $R_0 = N\frac{W-1/\tau}{W+1/\tau}$ characterizes the pump intensity; N is the density of atoms of the active substance; W is the probability of excitation under the action of the pump radiation; τ is the lifetime of the excited state; $T_1 = \tau/(1+W\tau)$, T_2^{-1} is the width of the emission line; $\hbar\omega_{12}$ is the energy difference between the upper and lower levels; d_{12} is the matrix element of the dipole moment.

When the field and polarization in (1) are expanded in terms of the eigenmodes of the field, it turns out that the expansion for the number of particles does not reduce to a division into two ensembles of particles, each of which interacts with only one type of oscillation. This is connected with the nonlinear dependence of the population of the energy levels on the field. Substituting R from (1c) into (1b), we obtain an equation containing only the polarization and the field and which can be expanded in the eigenfunctions of the field.

The equation obtained in this way is equivalent to the system*

$$\ddot{E}_i + \frac{\omega_i}{Q_i}\dot{E}_i + \omega_i^2 E_i = -4\pi\ddot{P}_i,$$

$$\ddot{P}_i + \frac{2}{T_2}\dot{P}_i + \omega_\ell^2 P_i = -\frac{2d_{12}\omega_{12}}{\hbar}\left[(R_0 + \mu_{iR}i + \mu_{12}R_j)E_i + \mu_{12}R_{12}E_j\right], \qquad (2)$$

$$\dot{R}_i + \frac{1}{T_1}R_i = \frac{2}{\hbar\omega_{12}}\left(\dot{P}_i + \frac{1}{T_2}P_i\right)E_i;$$

$$\dot{R}_{12} + \frac{1}{T_1}R_{12} = \frac{2}{\hbar\omega_{12}}\sum_{1,2}\left(\dot{P}_i + \frac{1}{T_2}P_i\right)E_j,$$

$$i \neq j; \qquad i,j = 1,2,$$

where E_i, P_i $(i = 1,2)$ are the field and polarization of the types of oscillations; R_1, R_2, R_{12} have the meaning of particles "illuminated" in the field; $\mu_i = \int \varphi_i^4\, dv$; $\mu_{12} = \int \varphi_1^2\varphi_2^2\, dv$** ($\varphi_i$ are the eigenfunctions of the resonator).

The character of the solution of system (2) depends on the mutual detuning of the types of oscillations. If the distance between the eigenfrequencies is considerably greater than the width of each resonance curve, the oscillations may occur simultaneously at both frequencies; but if the resonance curves substantially overlap, then one should expect the oscillations to be close to harmonic. Let us consider these two cases separately.

1. If the oscillations in the quantum generator are close to biharmonic, then in the practically interesting case of large times T_1 the system (2) can be simplified. The term R_{12} varies with the difference frequency $\omega_2 - \omega_1$, and for $(\omega_2 - \omega_1)T_1 \gg 1$ it is negligibly small in comparison with R_1 and R_2. For ruby, for example, $T_1 \sim 10^{-3}$–10^{-4} sec, and for the nearest types of oscillations with different axial numbers $\omega_2 - \omega_1 \sim c/L$ (L is the length of the resonator) and $cT_1/L \gg 1$. Therefore the interaction of types of oscillations differing in axial indices is, with good accuracy, described by equations (2) with $R_{12} = 0$.

In optical quantum generators the width of the radiation line of the substance is usually much smaller than the radiation frequency, and the width of the resonance curve of each type of oscillation is much smaller than the width of the spectral line. In this case, if the self-excitation conditions are not exceeded too strongly, from the Van der Pol equations for (2) one can obtain the system

$$\dot{U}_1 = 2\left[k\gamma_1(1 + \mu_1 R_1 + \mu_{12} R_2) - h_1\right] U_1,$$

$$\dot{U}_2 = 2\left[k\gamma_2(1 + \mu_2 R_2 + \mu_{12} R_1) - h_2\right] U_2, \tag{3}$$

$$\dot{R}_1 = -h\left[\gamma_1(1 + \mu_1 R_1 + \mu_{12} R_2)U_1 + R_1\right],$$

$$\dot{R}_2 = -h\left[\gamma_2(1 + \mu_2 R_2 + \mu_{12} R_1)U_2 + R_2\right].$$

Here

$$2h_i = \omega_i/\omega_\ell Q_i, \qquad 2k = 4\pi R_0 d_{12}^2 \hbar^{-1} T_2,$$

$$h_\ell^{-1} = \omega_\ell T_2, \qquad h^{-1} = \omega_\ell T_1, \qquad \gamma_i^{-1} = 1 + \delta_i^2/h_\ell^2,$$

$$\delta_i = (\omega_\ell - \omega_i)/\omega_\ell, \qquad i = 1, 2.$$

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* For large excesses of the pump power over the threshold, in expanding the polarization in the eigenfunctions of the resonator it is necessary to take into account a larger number of terms.

** It is assumed that $\mu_{13} = \int \varphi_1 \varphi_2^3 \, dv = \int \varphi_2 \varphi_1^3 \, dv = 0$.

In equations (3), the normalized variables U_i, R_i are proportional to the field energy of the modes and to the number of active particles. Equations of this kind are often called rate equations. Equations analogous in meaning were obtained in ([7]).

If the condition $1/T_1 \gg \omega_i/Q_i$, characteristic of quantum generators in gases, is satisfied, then system (3) is simplified further. At small saturations the resulting equations coincide with the equations studied in ([8]).

System (3) has four equilibrium states.

a) $U_i = R_i = 0$. The oscillations will grow if the excitation conditions are satisfied: $k\gamma_1 > h_1$ or $k\gamma_2 > h_2$. The excitation conditions for each mode are independent of one another.

b) $U_1 > 0,\ U_2 = 0$. The equilibrium state corresponds to the case in which only the first mode is excited. If the modes are symmetrically detuned relative to the center of the emission line ($\gamma_1 = \gamma_2 = \gamma$), then the excitation condition for the second mode has the form

$$k\gamma > \frac{h_2\mu_1 - h_1\mu_{12}}{\mu_1 - \mu_{12}}. \tag{4}$$

If the quality factor of the second mode is equal to the quality factor of the first, then oscillations in it will always grow; but if its quality factor is smaller, they may fail to be excited. Analogous reasoning applies for $U_2 > 0,\ U_1 = 0$. Thus a state is possible in which the excitation conditions for both modes are satisfied, but only the mode with the largest quality factor is excited.

c) $U_1 > 0,\ U_2 > 0$. Investigation of the characteristic equation shows that this equilibrium state is always stable.

After the transient process the system arrives at one of the possible equilibrium states. In any of them the radiation energy of the quantum generator is constant.

2. Let us consider two closely spaced modes. In this case the solution of system (2) is close to harmonic. Since it is assumed that the linewidth of the radiation of the substance is much greater than the width of the resonance curve of each of the modes, the Van der Pol equations for (2) can be used to obtain equations of "slow" motion ([9]). In what follows, for simplicity, we shall consider the case where $h_1 = h_2,\ \delta_1 = -\delta_2$. This means that the modes have the same quality factor and are symmetrically detuned relative to the center of the emission line. If, in addition, $\int \varphi_1^4\,d\nu = \int \varphi_2^4\,d\nu = \mu$, then the behavior of system (2) in the region of "slow" motions is described by the equations

$$\dot{x}_i = [k(1 + \mu R_i + \mu_{12}R_j) - h_1]x_i + x_j k R_{12}\mu_{12} - \delta_i y_i,$$

$$\dot{y}_i = [k(1 + \mu R_i + \mu_{12}R_j) - h_1]y_i + y_j k R_{12}\mu_{12} + \delta_i x_i, \tag{5}$$

Figure 1

Figure 1: Figure 1

$$\dot{R}_i = -h\{R_i + (1 + \mu R_i + \mu_{12} R_j)(x_i^2 + y_i^2) + (x_1 x_2 + y_1 y_2) R_{12} \mu_{12}\},$$

$$\dot{R}_{12} = -h\{R_{12} + [2 + (\mu + \mu_{12}) R_1 + (\mu + \mu_{12}) R_2](x_1 x_2 + y_1 y_2) +$$

$$+ \mu_{12} R_{12}(x_1^2 + y_1^2 + x_2^2 + y_2^2)\},$$

$$i \neq j; \quad i = 1, 2.$$

The normalized variables x_1, y_1, x_2, y_2 are the Van der Pol variables for E_1 and E_2, respectively. The values of the constant coefficients are the same as in (3). All limit cycles of the original equations (2)

and their stability are completely determined by the equations of "slow" motions (5). From the compatibility condition for system (5) it follows that stationary harmonic oscillations are possible only at frequencies $\omega = \omega_1$, $\omega = \omega_l$, $\omega = \omega_2$. If the oscillation frequency coincides with the natural frequency of the first type, then the stationary state is such that only this type of oscillation is excited, while the field in the second is equal to zero. For equal Q-factors of the types of oscillations, such a stationary regime is unstable. Oscillations will always also be excited in the second type. Analogously unstable is the state of equilibrium in which $\omega = \omega_2$, and the field is different from zero only in the second type of oscillation.

Fig. 1. Time dependence of the radiation intensity of an optical quantum generator in the region of instability of the harmonic regime. $\omega_l = 2.5 \cdot 10^{15}$ sec^{-1}, $T_1 = 4 \cdot 10^{-4}$ sec, $Q = 3 \cdot 10^6$, $(\omega_2 - \omega_1)\omega^{-1} Q = 1 \cdot 10^{-3}$. Excess above threshold 40%. As the unit the stationary value of the energy at $\Delta\omega = 0$ is taken.

If the oscillations occur at the line frequency, then the energy is distributed equally between both types of oscillations. This state of equilibrium is stable when the detuning of the types of oscillations relative to one another is sufficiently small. When the detuning $\Delta\omega = \omega_2 - \omega_1$ is increased to a critical value, which depends on the pump power, the stability of the harmonic regime is lost. Numerical integration of system (5) for various values of detuning and pump intensity shows that, after the transient process, a stationary solution is established which is independent of the choice of initial conditions. The stationary solution of system (5) in the region of instability can have a sharply expressed oscillatory character.

Figure 1 shows the time dependence of the energy of the quantum generator. The solution was obtained for the parameter values: $\omega_l = 2.5 \cdot 10^{15}$ sec^{-1}, $T_1 = 4 \cdot 10^{-4}$ sec, $Q = 3 \cdot 10^6$; excess above threshold 40%. The energy of the quantum generator has the form of undamped oscillations with period $T = 4 \cdot 10^{-6}$ sec and a sufficiently large modulation depth. With increasing pump intensity, the oscillation period decreases. At fixed pump power, one can pass from the self-modulation regime to a regime with constant-in-time energy of the quantum generator both by decreasing the relative detuning $\Delta\omega$ and by increasing it. In the first case stable harmonic oscillations are established in the system, and in the second—stable biharmonic oscillations approximately described by equations (3).

The regular undamped oscillations of the radiation intensity of an optical quantum generator that have been obtained explain certain regimes observed experimentally. In experiment, however, the picture is more complex, and therefore it is of interest to consider the interaction of a larger number of types of oscillations.

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