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Abstract

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DOUBLE INVARIANCE OF AUTOMATIC CONTROL SYSTEMS

Let an automatic control system be described by the equations

$$\begin{aligned} a_{11}(D,t)x_1 + a_{12}(D,t)x_2 + \dots + a_{1m}(D,t)x_m &= a_{f_1}(D,t)f_1, \\ a_{21}(D,t)x_1 + a_{22}(D,t)x_2 + \dots + a_{2m}(D,t)x_m &= a_{f_2}(D,t)f_2, \\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{m1}(D,t)x_1 + a_{m2}(D,t)x_2 + \dots + a_{mm}(D,t)x_m &= a_{fm}(D,t)f_m, \end{aligned} \quad (1)$$

where $x_1, x_2, \dots, x_m$ are the generalized coordinates of the system; $f_1, f_2, \dots, f_m$ are disturbing and control actions, some of which may consist of several identical terms, and we assume that $f_1, f_2, \dots, f_m$ are analytic functions bounded in modulus, holomorphic at $t = t_0$; $a_{ij}(D,t)$, $a_{fi}(D,t)$, $i,j = 1,2,\dots,m$, are second-order operators in $D = d/dt$ with variable coefficients. Some of them contain variable parameters of the plant.

Without reducing generality, suppose that we are interested in the invariance of the coordinate x_1 with respect to certain M actions among $f_1, f_2, \dots, f_m$. By transformations that do not violate the equivalence of system (1) and the new form of its notation ([1]), we reduce (1) to a single equation of the form

$$A(D,t)x_1 = \sum_{i=1}^{M} A_{f_i}(D,t)f_i, \quad (2)$$

where, in multichannel systems, $M \le m$ (in single-channel systems $M \le m-2$ ([2])). For $M < m$, the numbering of f_i in (2) and (1) may be different, i.e., it is not necessarily only the first M of the $f_1, f_2, \dots, f_m$ actions in (1) that are considered.

We shall call an automatic control system a **system possessing double invariance** if, for it, the conditions of invariance with respect to the actions $f_i(t)$ are simultaneously conditions of invariance of the operator $A(D,t)$ with respect to the plant parameters, i.e., if from the conditions of invariance of the system with respect to $f_i(t)$ it follows, as a consequence, that the operator $A(D,t)$ is independent of the plant parameters.

Double invariance obviously ensures zero sensitivity of the system with respect to the plant parameters. In the general case one may assert that the problem of insensitivity is the problem of invariance with respect to the system parameters.

Let us show that self-adjusting searchless systems with a model ([3],[4]), in the regime when the plant parameters, having changed in an arbitrary manner, then remain constant, possess double invariance. Indeed, let the equations of the system be ([4]):

$$\sum_{\alpha=0}^{k} a_\alpha^*(t)\varphi^{(i)} = -\,b^*(t)\mu \quad \text{—plant;}$$

$$\sum_{\gamma=0}^{r} c_\gamma \mu^{(\gamma)} = \sigma, \quad r+k=n, \quad \text{—actuator;}$$

$$\sigma = k_b\left(\sum_{i=0}^{n-1} k_i\varphi^{(i)} - k_g og\right) \quad \text{—control law;}$$

$$\left.\begin{aligned} k_i &= \bar{k}_i + k_{iu} z_i, \\ z_i &= \int_0^t \left(\varphi^{(i)} - \varphi_{\text{M}}^{(i)}\right)\Phi\left(\varphi_{\text{M}}^{(i)}\right)\operatorname{sign}\varphi^{(i)}\,dt, \\ \Phi\left(\varphi_{\text{M}}^{(i)}\right) &= \begin{cases} 1, & \text{for } |\varphi_{\text{M}}^{(i)}| > \Delta_i, \\ 0, & \text{for } |\varphi_{\text{M}}^{(i)}| \le \Delta_i, \end{cases} \\ \sum_{\xi=0}^{n} d_\xi \varphi_{\text{M}}^{(\xi)} &= g \end{aligned}\right\}\text{—law of variation of the adjustable coefficients,} \tag{3}$$

where φ is the controlled coordinate; φ_{M} is the model coordinate; μ is the coordinate of the regulating element; $a_\alpha^*(t)$, $b^*(t)$ are the coefficients of the plant; c_γ, $\bar{k}_i$, k_{iu}, Δ_i, d_ξ, k_{g0} are constant coefficients; k_b is the overall controller gain; g is the control action.

System (3), for $z_i = \tilde{z}_i + \Delta z_i$, $\tilde{z}_i = z_{i\,\text{st}} = \text{const}$, $|\varphi_{\text{M}}^{(i)}| > \Delta_i$, $\Delta z_i = D^i\Delta z_0 \operatorname{sign} D^i(\varepsilon+\varphi_{\text{M}})\operatorname{sign}(\varepsilon+\varphi_{\text{M}})$, $i=1,2,\ldots,n-1$, $\varepsilon = \varphi - \varphi_{\text{M}}$, reduces to a single equation

$$\left\{D^{n+1} + \sum_{i=0}^{n-1}(a_i + \varkappa\bar{k}_i + \varkappa k_{iu}\tilde{z}_i)D^{i+1} + \varkappa\sum_{i=0}^{n-1} k_{iu}(D^i\varphi_{\text{M}})\operatorname{sign}D^i\varphi_{\text{M}}\cdot D^i\right\}\Delta z_0 =$$

$$= -\left\{(1 - k_{g0}\varkappa d_n)D^n + \sum_{i=0}^{n-1}(a_i + \varkappa\bar{k}_i + \varkappa k_{iu}\tilde{z}_i - \varkappa k_{g0}d_i)D^i\right\}\varphi_{\mathrm{M}}\,\mathrm{sign}\,\varphi_{\mathrm{M}}, \quad (4)$$

where $\varkappa = k_b b^*/c_r a_k^* = \mathrm{const}$ (see (4)), and a_i are the coefficients of the polynomial

$$\sum_{i=0}^{n} a_i D^i = \frac{1}{c_r a_k^*}\sum_{\alpha=0}^{k} a_\alpha^* D^\alpha \sum_{\gamma=0}^{r} c_\gamma D^\gamma. \quad (5)$$

When the invariance conditions $\varepsilon = \varphi - \varphi_{\mathrm{M}}$ with respect to g are satisfied (conditions (10) in (4)),

$$d_n = \frac{1}{\varkappa k_{g0}}, \qquad \tilde{z}_i = \frac{\varkappa k_{g0}d_i - \varkappa\bar{k}_i - a_i}{\varkappa k_{iu}}, \quad (6)$$

equation (4) can be written in the form

$$\Big(D^{n+1} + \varkappa k_{g0}d_{n-1}D^n + (\varkappa k_{g0}d_{n-2} + \varkappa k_{n-1,u}|\varphi_{\mathrm{M}}^{(n-1)}|)D^{n-1} + \ldots$$

$$\ldots + (\varkappa k_{g0}d_0 + \varkappa k_{1u}|\varphi_{\mathrm{M}}|)D + \varkappa k_{0u}|\varphi_{\mathrm{M}}|\Big)\,\Delta z_0 = 0. \quad (7)$$

It follows from this that, when (6) is satisfied, the operator $A(D,t)$ of order $(n+1)$ of equation (4) does not depend on the plant parameters a_α^*, which is what was required to be proved.

Let us note that the systems considered above in the indicated mode are also invariant with respect to the coefficients of the actuating mechanism c_γ and the constant coefficients $\bar{k}_i$, $i = 0, 1, \ldots, n = 1$.

It can be shown that, for real differentiating circuits, double invariance of searchless self-adjusting systems with a model is fulfilled only with respect to the coefficients of the operator of order $2n$, $A(D,t)$, at $D^0, D, D^2, \ldots, D^n$. The coefficients before $D^{n+1}, D^{n+2}, \ldots, D^{2n}$ include a_α, but multiplied by the intrinsic time constants of the differentiating circuits, which are small quantities. On the right-hand side of (7) there appear terms with $\varphi_{\mathrm{M}}^{(2n-1)}, \ldots, \varphi_{\mathrm{M}}^{(n+1)}$, whose coefficients are likewise small quantities. In this case one may speak of partial double invariance of system (5).

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.