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Soviet-era science, translated into English

# L. N. Shevrin

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## Abstract

## Full Text

## LETTER TO THE EDITOR

In my article “On densely embedded ideals of semigroups,” published in *DAN*, vol. 131, No. 4, 1960, in the proof of the theorem on p. 766 there is an inaccuracy: in the proof of Lemma 3 the equalities  $K_{\alpha\beta\gamma} = a_{\tau(\alpha,\gamma)}$  and  $\alpha_\beta K_{\gamma\delta} = a_{\tau(\beta,\delta)}$  will be valid only under the condition that certain complexes  $K_{\xi\gamma}$  and  $K_{\beta\eta}$  are nonempty, which, generally speaking, cannot be guaranteed.

This defect can easily be eliminated, and, in essence, within the same construction. Namely, preserving the notation of the note, we construct the semigroup  $S = \{T, b\}$ , in which, along with all defining relations holding in  $T$ , we require the following defining relations to hold:  $b^2 = a$ ;  $b0 = 0$ ; for any  $t_1, t_2 \in T$ ,  $bt_1b = t_1bt_2 = 0$ , and, finally,  $bt_1 = bt_2$  ( $t_1b = t_2b$ ) if and only if  $at_1 = at_2$  ( $t_1a = t_2a$ ). It is easy to see that, in this case, no “gluings” occur in  $T$ , and, moreover,  $T$  is properly contained in  $S$ , since in every case  $b \in S \setminus T$ . From the fact that  $N$  is an ideal in  $T$  and  $bN = Nb = 0$ , it follows that  $N$  will be an ideal in  $S$ . Next, we shall show that every true homomorphism  $\varphi$  of the semigroup  $S$  induces a true homomorphism on  $N$ . Let  $\varphi x = \varphi y$ ,  $x \neq y$ ,  $x, y \in S$ . It is clear that it suffices to consider only the case when at least one of the elements  $x, y$  does not belong to  $T$ ; without loss of generality we assume that  $x \in T$ . The proof that  $\varphi$  induces a true homomorphism of  $T$  (and therefore also of  $N$ ) is almost a verbatim repetition of the concluding part of the proof of the theorem of the note. Let us examine, for example, one of the possible cases  $x = bt$ ,  $t \in T$  (in the other cases  $x = b$  and  $x = tb$ , the arguments are likewise reworked from the corresponding arguments of the note in an analogous manner). First of all, note that, by the defining relations,  $at \neq 0$ : from  $at = 0 = a0$  it would follow that  $bt = b0 = 0 \in T$ . The following equalities hold:

$$\varphi at = \varphi by, \quad (1)$$

$$\varphi ay = \varphi 0. \quad (2)$$

Indeed,  $\varphi at = \varphi bbt = \varphi b \cdot \varphi x = \varphi b \cdot \varphi y = \varphi by$  and  $\varphi ay = \varphi a \cdot \varphi y = \varphi a \cdot \varphi x = \varphi abt = \varphi 0$ . We shall show that there exist  $u, v \in T$  such that  $u \neq v$  and  $\varphi u = \varphi v$ . Consider the possibilities that may occur.

1.  $y \in T$ . If  $ay = 0$ , then  $by = 0$ , and what is required follows from equality (1). If, however,  $ay \neq 0$ , then what is required is provided by equality (2).
2.  $y \in S \setminus T$ .

2.1.  $y = b$ . Here, taking equality (1) into account, we obtain  $\varphi at = \varphi yy = (\varphi y)^2 = \varphi x \cdot \varphi y = \varphi xy = \varphi btb = \varphi 0$ .

2.2.  $y = bt_1$ . Here  $at \neq at_1$  (since  $x \neq y$ ) and, by (1),  $\varphi at = \varphi bbt_1 = \varphi at_1$ .

2.3.  $y = t_2b$ . Here, by (1),  $\varphi at = \varphi bt_2b = \varphi 0$ .

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*Note: Figure translations are in progress. See original paper for figures.*

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