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Abstract

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SPHERICAL ABERRATION OF A COMBINED QUADRUPOLE LENS WITH A RECTANGULAR FIELD DISTRIBUTION

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In commonly used quadrupole lenses the length of the poles is several times greater than the aperture radius. In this case the dependence of the field on the coordinate z (the z-axis coincides with the axis of the lens) is a plateau with sharply cut-off edges. Approximation of a field of this kind by a rectangle greatly simplifies the calculation of first-order properties and at the same time gives results in good agreement with experiment.

Of great interest is the calculation of the spherical aberration of this type of field. However, an attempt to use the rectangular model in calculating spherical aberration encounters difficulties. They are associated with the fact that the field derivatives entering the trajectory equations, which include third-order terms (see ([1])), become infinitely large at the edges of the lens. In order to avoid this difficulty, we have obtained expressions for spherical aberration that do not explicitly contain field derivatives. These expressions were found by solving, by means of the perturbation method, the trajectory equations given in ([1]), and by a subsequent transformation of the formulas obtained, using integration by parts. In the present work we consider a field bounded in the axial direction, in the case where a point object lying on the axis and its linear image are located in field-free space. The expressions for the spherical aberration of a combined quadrupole lens in this case have the form

$$\Delta x_C = {}^1\!/_2 x_0' M_C \Bigg\{ x_0'^2 \beta^2 \left[\int_0^L f(z) x_\gamma^2 x_\gamma {a'}^2\, dz + {}^1\!/_3 \beta^2 (2 - 2n + 3n^2) \int_0^L f^2(z) x_\gamma^4\, dz \right] + y_0'^2 \left[3a + \frac{1}{M_C^2 M_D^2}(2g_C + g_D) + 3\int_0^L x_\gamma {a'}^2 y_\gamma {a'}^2\, dz + \beta^4 (2 + 2n - n^2) \int_0^L f^2(z) x_\gamma^2 y_\gamma^2\, dz \right] \Bigg\}; \quad (1)$$

$$\Delta y_D = {}^1\!/_2 y_0' M_D \Biggl\{ y_0'^2 \beta^2 \left[-\int_0^L f(z) y_\gamma^2 y_\gamma a'^2\, dz + {}^1\!/_3 \beta^2 (2 - 2n + 3n^2) \int_0^L f^2(z) y_\gamma^4\, dz \right] + x_0'^2 \left[3a + \frac{1}{M_C^2 M_D^2}(g_C + 2g_D) + 3 \int_0^L x_\gamma a'^2 y_\gamma a'^2\, dz + \beta^4 (2 + 2n - n^2) \int_0^L f^2(z) x_\gamma^2 y_\gamma^2\, dz \right] \Biggr\}. \tag{2}$$

The x and y axes are arranged so that the lens is converging in the xz plane and diverging in the yz plane. Δx_C and Δy_D determine the spherical aberration of the width of the linear image (real or virtual) lying in the planes $x = 0$ and $y = 0$, respectively. In (1) and (2) the following notation has been introduced: $f(z)$ is the dependence of the field distribution on the coordinate z; x_γ and y_γ are solutions of the paraxial trajectory equations satisfying the following conditions at the entrance to the field:

$$x_{\gamma 0} = y_{\gamma 0} = a, \qquad x'_{\gamma 0} = y'_{\gamma 0} = 1. \tag{3}$$

Then the paraxial trajectory has the form

$$x = x_0' x_\gamma, \qquad y = y_0' y_\gamma, \tag{4}$$

where x_0' and y_0' determine the slope of the trajectory in object space, M_C and M_D are the magnifications in the converging and diverging planes, respectively; L is the length of the field; a is the distance from the object to the entrance into the field; g_C and g_D are the distances from the exit from the field to the image in the converging and diverging planes. Positive a or g correspond to a real object or image, negative ones to a virtual one. The excitation of the combined lens is denoted by $\beta^2 = \beta_M^2 - \beta_E^2$, where β_M^2 and β_E^2 are the excitations of its magnetic and electric lens components. The ratio between the electric and magnetic fields for a given β^2 is determined by the coefficient n.

$$\beta_E^2 / \beta_M^2 = n/(n+1). \tag{5}$$

Let us integrate expressions (1) and (2) for a field of rectangular form (over the whole extent of the field $f(z) = 1$). We obtain

$$\Delta x_C = M_C x_0' (C_p x_0'^2 + C_s y_0'^2), \tag{6}$$

$$\Delta y_D = M_D y_0' (D_p y_0'^2 + D_s x_0'^2), \tag{7}$$

where

$$\frac{C_p}{L} = \frac{1}{16}\left\{(1+\beta^2a^2)^2 - (1-6\beta^2a^2+\beta^4a^4)\frac{\sin 4\beta L}{4\beta L}+\right.$$

$$+\frac{a}{L}(1-\beta^2a^2)(1-\cos 4\beta L)+\frac{1}{3}(2-2n+3n^2)\times$$

$$\times\left[3(1+\beta^2a^2)^2-4(1-\beta^4a^4)\frac{\sin 2\beta L}{2\beta L}+(1-6\beta^2a^2+\beta^4a^4)\frac{\sin 4\beta L}{4\beta L}+\right.$$

$$\left.\left.+4\frac{a}{L}(1+\beta^2a^2)(1-\cos 2\beta L)-\frac{a}{L}(1-\beta^2a^2)(1-\cos 4\beta L)\right]\right\}; \qquad (8)$$

$$\frac{C_s}{L} = \frac{1}{16}\left\{6(1-\beta^4a^4)+2\frac{a}{L}\left[3-(1-\beta^2a^2)\cos 2\beta L+(1+\beta^2a^2)\operatorname{ch} 2\beta L-\right.\right.$$

$$\left.-(2-3\beta^2a^2)\sin 2\beta L\ \operatorname{sh} 2\beta L-(3+2\beta^2a^2)\cos 2\beta L\ \operatorname{ch} 2\beta L\right]-$$

$$-\left[2(1-\beta^2a^2)^2+(5-4\beta^2a^2-5\beta^4a^4)\operatorname{ch} 2\beta L\right]\frac{\sin 2\beta L}{2\beta L}+$$

$$\left.+\left[2(1+\beta^2a^2)^2-(1+20\beta^2a^2-\beta^4a^4)\cos 2\beta L\right]\frac{\operatorname{sh} 2\beta L}{2\beta L}+\varphi(n)\right\}; \qquad (9)$$

$$\frac{D_p}{L} = \frac{1}{16}\left\{(1-\beta^2a^2)^2-(1+6\beta^2a^2+\beta^4a^4)\frac{\operatorname{sh} 4\beta L}{4\beta L}+\right.$$

$$+\frac{a}{L}(1+\beta^2a^2)(1-\operatorname{ch} 4\beta L)+\frac{1}{3}(2-2n+3n^2)\times$$

$$\times\left[3(1-\beta^2a^2)^2-4(1-\beta^4a^4)\frac{\operatorname{sh} 2\beta L}{2\beta L}+(1+6\beta^2a^2+\beta^4a^4)\frac{\operatorname{sh} 4\beta L}{4\beta L}+\right.$$

$$\left.\left.+4\frac{a}{L}(1-\beta^2a^2)(1-\operatorname{ch} 2\beta L)-\frac{a}{L}(1+\beta^2a^2)(1-\operatorname{ch} 4\beta L)\right]\right\}; \qquad (10)$$

$$\frac{D_s}{L} = \frac{1}{16}\left\{6(1-\beta^4a^4)+2\frac{a}{L}\left[3+(1-\beta^2a^2)\cos 2\beta L-(1+\beta^2a^2)\operatorname{ch} 2\beta L+\right.\right.$$

$$\left.+(2+3\beta^2a^2)\sin 2\beta L\ \operatorname{sh} 2\beta L-(3-2\beta^2a^2)\cos 2\beta L\ \operatorname{ch} 2\beta L\right]+$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$+\left[2(1-\beta^2a^2)^2-(1-20\beta^2a^2-\beta^4a^4)\operatorname{ch}2\beta L\right]\frac{\sin 2\beta L}{2\beta L}-$$

$$-\left[2(1+\beta^2a^2)^2+(5+4\beta^2a^2-5\beta^4a^4)\cos 2\beta L\right]\frac{\operatorname{sh}2\beta L}{2\beta L}+\varphi(n)\Bigr\}; \qquad (11)$$

$$\varphi(n)=(2+2n-n^2)\Bigl\{-2(1-\beta^4a^4)-2\frac{a}{L}\left[1-(1-\beta^2a^2)\cos 2\beta L-\right.$$

$$\left.-(1+\beta^2a^2)\operatorname{ch}2\beta L-\beta^2a^2\sin 2\beta L\ \operatorname{sh}2\beta L+\cos 2\beta L\ \operatorname{ch}2\beta L\right]+$$

$$+\left[2(1-\beta^2a^2)^2-(1-4\beta^2a^2-\beta^4a^4)\operatorname{ch}2\beta L\right]\frac{\sin 2\beta L}{2\beta L}+$$

$$+\left[2(1+\beta^2a^2)^2-(1+4\beta^2a^2-\beta^4a^4)\cos 2\beta L\right]\frac{\operatorname{sh}2\beta L}{2\beta L}\Bigr\}. \qquad (12)$$

At high magnifications (the object is located at the focal point of the converging plane)

$$\beta a=\operatorname{ctg}\beta L, \qquad (13)$$

Fig. 1. Coefficients of spherical aberration C_p for the rectangular field model in the case of high magnifications: solid curves; and in the short-lens approximation: dashed curves. 1 and 1a—magnetic lens, 2 and 2a—electric lens, 3—achromatic.

Fig. 2. Coefficients of spherical aberration D_p. The notation is the same as in Fig. 1.

and the expressions for C_p, C_s, D_p, and D_s are considerably simplified:

$$\frac{C_p}{L}=\frac{1}{16\sin^4\beta L}\left[1-\frac{\sin 4\beta L}{4\beta L}+\frac{1}{3}(2-2n+3n^2)\left(3+4\frac{\sin 2\beta L}{2\beta L}+\frac{\sin 4\beta L}{4\beta L}\right)\right]; \qquad (14)$$

$$\frac{D_p}{L}=\frac{1}{16\sin^4\beta L}\left\{\frac{1+\cos 4\beta L}{2}+(1-\operatorname{ch}4\beta L)\frac{\sin 2\beta L}{2\beta L}-\frac{3-\cos 4\beta L}{2}\frac{\operatorname{sh}4\beta L}{4\beta L}+\right.$$
$$+\frac{1}{3}(2-2n+3n^2)\left[\frac{3(1+\cos 4\beta L)}{2}-(1-\operatorname{ch}4\beta L)\frac{\sin 2\beta L}{2\beta L}-\right.$$
$$\left.\left.-4(1-\operatorname{ch}2\beta L)\frac{\sin 4\beta L}{4\beta L}+4\cos 2\beta L\frac{\operatorname{sh}2\beta L}{2\beta L}+\frac{3-\cos 4\beta L}{2}\frac{\operatorname{sh}4\beta L}{4\beta L}\right]\right\}; \quad (15)$$

$$\frac{C_s}{L}=\frac{D_s}{L}=\frac{1}{16\sin^4\beta L}\left\{3\left[-2\cos 2\beta L+(1+\operatorname{ch}2\beta L)\frac{\sin 2\beta L}{2\beta L}-\frac{\sin 4\beta L}{4\beta L}+\right.\right.$$
$$\left.+\frac{\operatorname{sh}2\beta L}{2\beta L}\right]+(2+2n-n^2)\left[2\cos 2\beta L-(1-3\operatorname{ch}2\beta L)\frac{\sin 2\beta L}{2\beta L}+\right.$$
$$\left.\left.+\frac{\sin 4\beta L}{4\beta L}+3\frac{\operatorname{sh}2\beta L}{2\beta L}\right]\right\}. \quad (16)$$

The equality $C_s = D_s$ in the case of high magnifications is valid not only for the rectangular model, but also for any field distribution. The dependence of the spherical-aberration coefficients on the value βL for a rectangular field is given in Figs. 1, 2, and 3. In these same figures, for comparison, the coefficients calculated in the short-lens approximation are also shown:

$$\frac{C_p}{L}=\frac{D_p}{L}=\frac{1}{6}(2-2n+3n^2)\frac{1}{(\beta L)^4};\qquad \frac{C_s}{L}=\frac{D_s}{L}=\frac{1}{2}(2+2n-n^2)\frac{1}{(\beta L)^4}. \quad (17)$$

The last formula does not include the values $n = 1\pm\sqrt{3}$ (see (2)). In the case of a parallel beam incident on the lens, the expressions for the spherical aberration take the form:

$$\Delta x_{C\|}=\frac{x_0}{L}\left[C_{p\|}\left(\frac{x_0}{L}\right)^2+C_{s\|}\left(\frac{y_0}{L}\right)^2\right]; \quad (18)$$

$$\Delta y_{D\|}=\frac{y_0}{L}\left[D_{p\|}\left(\frac{y_0}{L}\right)^2+D_{s\|}\left(\frac{x_0}{L}\right)^2\right], \quad (19)$$

$$\frac{C_{p\|}}{L}=-\frac{(\beta L)^3}{16\sin\beta L}\left[1-\frac{\sin 4\beta L}{4\beta L}+\frac{1}{3}(2-2n+3n^2)\left(3+4\frac{\sin 2\beta L}{2\beta L}+\frac{\sin 4\beta L}{4\beta L}\right)\right];$$

$$\frac{C_{s\|}}{L}=-\frac{(\beta L)^3}{16\sin\beta L}\left[-6-(2-5\operatorname{ch}2\beta L)\frac{\sin 2\beta L}{2\beta L}+(2+\cos 2\beta L)\frac{\operatorname{sh}2\beta L}{2\beta L}+\varphi_\|(n)\right];$$

Fig. 3. Coefficients of spherical aberration $C_s = D_s$. For notation see Fig. 1

Figure 3: Fig. 3. Coefficients of spherical aberration $C_s = D_s$. For notation see Fig. 1

$$\frac{D_{p\|}}{L} = \frac{(\beta L)^3}{16\,\mathrm{sh}\,\beta L}\left[1 - \frac{\mathrm{sh}\,4\beta L}{4\beta L} + \frac{1}{3}(2-2n+3n^2)\left(3 + 4\frac{\mathrm{sh}\,2\beta L}{2\beta L} + \frac{\mathrm{sh}\,4\beta L}{4\beta L}\right)\right];$$

$$\frac{D_{s\|}}{L} = \frac{(\beta L)^3}{16\,\mathrm{sh}\,\beta L}\left[-6 + (2+\mathrm{ch}\,2\beta L)\frac{\sin 2\beta L}{2\beta L} - (2 - 5\cos 2\beta L)\frac{\mathrm{sh}\,2\beta L}{2\beta L} + \varphi_{\|}(n)\right];$$

$$\varphi_{\|}(n) = (2+2n-n^2)\left[2 + (2+\mathrm{ch}\,2\beta L)\frac{\sin 2\beta L}{2\beta L} + (2+\cos 2\beta L)\frac{\mathrm{sh}\,2\beta L}{2\beta L}\right].$$

Fig. 3. Coefficients of spherical aberration $C_s = D_s$. For notation see Fig. 1.

They are obtained from (6)–(12) by passing to the limit as $a \to \infty$, taking into account that $x_0' a = x_0$ and $y_0' a = y_0$.

Let us compare the results presented for a parallel beam with experimental data for an electrostatic quadrupole lens [3]. In that work the maximum spherical aberration of the width of the linear image, $(\Delta x_C)_{\max}$, was measured. For an electrostatic lens $(\Delta x_C)_{\max}$ corresponds to the value $y_0 = 0$ ($x_0 = R_0$). From (18) it is seen that the dependence of $(\Delta x_C)_{\max}/R_0$ on R_0 is a parabola,

$$(\Delta x_C)_{\max}/R_0 = \alpha_T R_0^2.$$

The curve given in [3], $(\Delta x_C)_{\max}/R_0$ ($\beta L \simeq 0.72$), can also be well approximated by the parabola

$$(\Delta x_C)_{\max}/R_0 = \alpha_e R_0^2,$$

where $\alpha_e = 2\cdot 10^{-3}$ cm^{-2}. The value calculated by formula (18) for $\beta L = 0.72$ is $\alpha_T = 1.35\cdot 10^{-3}$ cm^{-2}. As can be seen, the experimental and calculated values of α agree fairly well with each other.

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Note: Figure translations are in progress. See original paper for figures.