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Abstract

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A LINEARIZED PROBLEM ON THE FLOW OF AN INCOMPRESSIBLE FLUID WITH JET SEPARATION PAST A PERMEABLE WEDGE

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Below we shall assume that the velocity, normal to the faces of the wedge, of the seepage of the fluid through the wedge is proportional to the pressure difference on the two sides of the wedge:

$$v_n = k(p - p_\infty), \tag{1}$$

where k is the seepage coefficient, which we regard as small (see Fig. 1). The region occupied by the moving fluid is mapped onto the upper half of the auxiliary u-plane ($\operatorname{Im} u \geq 0$) in such a way that the points A and B correspond on the u-plane to the values $u_A = -1$, $u_B = 1$, while infinitely distant points in the z- and u-planes pass into one another (see Fig. 2). In the u-plane the rays AC' and BC are streamlines, while along the segment AB sinks are continuously distributed, their strengths being determined as functions of the distribution of the velocity v_n on the wedge. The complex potential of the fictitious flow in the u-plane will be

$$W = Nu^2 - \frac{1}{\pi}\int_{-1}^{1} q(\xi)\ln(u - \xi)\,d\xi, \tag{2}$$

where $q(\xi)$ characterizes the strength of the sink at the point ξ. If φ is the velocity potential, ψ the stream function, n the direction along the normal to the faces of the wedge, and l the direction along the wedge, then we may write

$$v_n = \frac{\partial\varphi}{\partial n} = \frac{\partial\psi}{\partial l} = \frac{\partial\psi}{\partial u}\frac{du}{dl}.$$

But $\partial\psi/\partial u = q(u)$, $v_l = d\varphi/dl$, where v_l is the magnitude of the fluid velocity along the wedge. Hence

$$q(u) = \frac{v_n}{v_l}\frac{d\varphi}{dn}. \tag{3}$$

Fig. 1

Figure 1: Fig. 1

Fig. 1

The quantity $q(u)$ is small. Therefore, in determining $q(u)$ from (3), we retain small quantities of order k, i.e. we take $d\varphi/dn$ and v_l from the Kirchhoff flow past an impermeable wedge, and v_n from (1). The solution of the problem of flow past a wedge according to Kirchhoff can be brought to the form

$$w_0 = Nu^2, \qquad \frac{dw_0}{dz_0} = -V_\infty e^{i\chi\pi}\left(\frac{u}{1+\sqrt{1-u^2}}\right)^{2(1-\chi)}.$$

Hence

$$-\frac{d\varphi}{du} = 2Nu, \qquad v_l = V_\infty\left(\frac{u}{1+\sqrt{1-u^2}}\right)^{2(1-\chi)}. \tag{4}$$

Using Bernoulli' s integral and neglecting terms of order higher than k, we transform (1) to the form

$$v_n = \frac{k\rho}{2}\left(V_\infty^2 - v_l^2\right). \tag{5}$$

From (3), (4), and (5) we obtain

$$q'(\xi) = k\rho V_\infty N\xi\left[\left(\frac{\xi}{1+\sqrt{1-\xi^2}}\right)^{-2(1-\chi)} - \left(\frac{\xi}{1+\sqrt{1-\xi^2}}\right)^{2(1-\chi)}\right]; \tag{6}$$

from (2) and (6)

$$\frac{dW}{du} = 2Nu\left[1 - \frac{k\rho V_\infty}{\pi}\int_0^1 \frac{\left(\xi/(1+\sqrt{1+\xi^2})\right)^{-2(1-\chi)} - \left(\xi/(1+\sqrt{1-\xi^2})\right)^{2(1-\chi)}}{u^2-\xi^2}\,\xi\, d\xi\right]. \tag{7}$$

Introduce the functions

$$\Omega = \ln\frac{dW}{V_\infty dz} = \ln\frac{V}{V_\infty} - i\theta, \qquad f(u) = \frac{\Omega(u)}{\sqrt{u^2-1}}. \tag{8}$$

Fig. 2: diagram of the u-axis with points C', A, 0, B, C marked at $-\infty$, -1, 0, $+1$, $+\infty$.

Figure 2: Fig. 2: diagram of the u-axis with points C', A, 0, B, C marked at $-\infty$, -1, 0, $+1$, $+\infty$.

On the free streamlines $V = V_\infty$, therefore for $1 \leq u \leq -1$, $\ln V/V_\infty = 0$, $\operatorname{Re} f(u) = 0$. On the segment $-1 < u < 1$

$$\operatorname{Re} f(u) = -\theta(u)/\sqrt{1-u^2}.$$

Determining, by means of Schwarz' s formula, $f(u)$ in the upper half-plane from its real part specified on the real axis, we find

$$f(u) = -\frac{1}{\pi i}\int_{-1}^{1}\frac{\theta(\xi)\,d\xi}{\sqrt{1-\xi^2}(\xi-u)}. \tag{9}$$

From Figs. 1 and 2 it is seen that

$$\theta(u) = \pi/2 - \pi\chi + \gamma(u), \qquad \operatorname{ctg}\gamma(u) = v_n/v_l. \tag{10}$$

Fig. 2

From (4), (5), (8), (9), and (10) we find:

$$\frac{dW}{V_\infty dz} = -e^{i\pi\chi}\left(\frac{u}{1+\sqrt{1-u^2}}\right)^{2(1-\chi)}\times$$

$$\times\left[1 - \frac{k\rho V_\infty\sqrt{1-u^2}}{\pi}\int_0^1\frac{\left(\xi/(1+\sqrt{1-\xi^2})\right)^{-2(1-\chi)} - \left(\xi/(1+\sqrt{1-\xi^2})\right)^{2(1-\chi)}}{\sqrt{1-\xi^2}\,(u^2-\xi^2)}\,\xi\,d\xi\right]. \tag{11}$$

Formulas (7) and (11) give the general solution of the problem.
For the vector $\overline{OB}$, from the basic equalities (7) and (11) we find

$$\overline{OB} = \int_0^1\frac{dz}{du}\,du = \frac{1}{V_\infty}\int_0^1\frac{V_\infty dz}{dW}\,\frac{dW}{du}\,du$$

or

$$\overline{OB} = -\frac{2N}{V_\infty e^{i\pi\chi}}\int_0^1 u\left(\frac{1+\sqrt{1-u^2}}{u}\right)^{2(1-\chi)}\left\{1+\frac{k\rho V_\infty}{\pi}\times\right. \tag{12}$$

$$\times \int_0^1 \frac{\left[(\xi/(1+\sqrt{1-\xi^2}))^{-2(1-\chi)} - (\xi/(1+\sqrt{1-\xi^2}))^{2(1-\chi)}\right] (\sqrt{1-u^2} - \sqrt{1-\xi^2})\, \xi\, d\xi}{\sqrt{1-\xi^2}\,(u^2-\xi^2)} \Bigg\} du.$$

Using the formula

$$X = -i\, \frac{\rho V_\infty^2}{2} \int^A dz^*,$$

where the asterisk means that the given quantity is expressed in terms of η by the formula $\eta = 1/u$ and by equalities (7) and (11), we find the drag of a symmetric permeable wedge with an arbitrary aperture angle in the form

$$X = \pi \rho V_\infty N \Bigg\{ 4(1-\chi)^2 + \frac{2k\rho V_\infty}{\pi} \times \tag{13}$$

$$\times \int_0^1 \frac{\left[(\xi/(1+\sqrt{1-\xi^2}))^{-2(1-\chi)} - (\xi/(1+\sqrt{1-\xi^2}))^{2(1-\chi)}\right] [\sqrt{1-\xi^2} - 2(1-\chi)]\, \xi\, d\xi}{\sqrt{1-\xi^2}} \Bigg\}.$$

Using (12) and (13), one can calculate the drag coefficient of a symmetric permeable wedge with an arbitrary aperture angle.

Let us consider some special cases.

1. If $k = 0$, then, by (7), (11), (12), and (13), we obtain the solution of the problem for an impermeable symmetric wedge.
2. For $k \neq 0$ and $\chi = 1/2$ we obtain the solution of the problem for a permeable symmetric plate

$$c_x = \frac{2X}{\rho V_\infty^2 H} = \frac{2\pi}{4+\pi} \left[1 + \frac{k\rho V_\infty}{\pi(4+\pi)} (8\pi - \pi^2 - 12) \right].$$

It is seen from this that for arbitrarily small k, $(c_x)_{k=0} < c_x$. At first glance this seems impossible; however, it becomes understandable if one considers the general picture of the flow.

From (11), setting $\chi = 1/2$, we obtain

$$\frac{dW}{V_\infty dz} = -\frac{iu}{1+\sqrt{1-u^2}} \left[1 + \frac{k\rho V_\infty}{\pi u} \sqrt{1-u^2} \ln \frac{1-u}{1+u} + i\, \frac{k\rho V_\infty}{u} \sqrt{1-u^2} \right].$$

Hence

$$\frac{v_x}{V_\infty} = \frac{k\rho V_\infty \sqrt{1-u^2}}{1+\sqrt{1-u^2}},$$

$$\frac{v_y}{V_\infty} = \frac{u}{1+\sqrt{1-u^2}} - \frac{k\rho V_\infty \sqrt{1-u^2}}{\pi(1+\sqrt{1-u^2})} \ln\frac{1+u}{1-u}, \qquad \text{for } 1 > u > 0,$$

$$\frac{v_y}{V_\infty} = \frac{u}{1+\sqrt{1-u^2}} + \frac{k\rho V_\infty \sqrt{1-u^2}}{\pi(1+\sqrt{1-u^2})} \ln\frac{1-u}{1+u}, \qquad \text{for } -1 < u < 0.$$

From these formulas it is seen that the velocity of the perturbed flow on the plate is always directed opposite to the velocity of the main stream. Therefore the velocity of the fluid at the permeable plate decreases; accordingly, by Bernoulli' s integral, the pressure increases, and hence the drag coefficient also—

increases. At the same time, the velocity component normal to the plate is a small quantity of the second order, which we neglect.

3. For $k \neq 0$ and $\chi = 1/4$, from (12) and (13) we obtain

$$\overline{OB} = -\frac{N}{V_\infty}(17{,}68 + 25{,}01k\rho V_\infty) + i\frac{N}{V_\infty}(17{,}68 + 25{,}01k\rho V_\infty),$$

$$X = \pi\rho V_\infty N(2{,}25 - 9{,}71k\rho V_\infty).$$

Hence

$$c_x = X\big/{}^1\!/_2\rho V_\infty^2 \cdot 2\ \mathrm{Im}\,\overline{OB} = 0{,}40 - 2{,}29k\rho V_\infty,$$

i.e., permeability decreases the drag coefficient of a symmetric wedge with opening angle $\pi/2$ and situated in the flow with its base forward.

4. For $k \neq 0$ and $\chi = 3/4$, from (12) and (13) we obtain

$$\overline{OB} = \frac{N}{V_\infty}(1{,}23 + 1{,}74k\rho V_\infty) + i\frac{N}{V_\infty}(1{,}23 + 1{,}74k\rho V_\infty),$$

$$X = \pi\rho V_\infty N(0{,}25 + 1{,}24k\rho V_\infty).$$

Hence

$$c_x = 0{,}63 + 2{,}26k\rho V_\infty,$$

i.e., permeability increases the drag coefficient of a symmetric wedge with opening angle $\pi/2$ and situated in the flow with its tip forward.

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CITED LITERATURE

1. M. I. Gurevich, *Prikl. matem. i mekh.*, No. 1 (1965).
2. M. I. Gurevich, *Theory of Jets of an Ideal Fluid*, Moscow, 1961.
3. I. S. Gradshtein, I. M. Ryzhik, *Tables of Integrals, Sums, Series, and Products*, Moscow, 1962.
4. M. T. Bekulov, *Proceedings of the Postgraduates of Kabardino-Balkarian State University*, issue 1, Nalchik, 1965.

Note: Figure translations are in progress. See original paper for figures.