REFLECTION AND REFRACTION OF PLANE PLASTIC WAVES

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

THEORY OF ELASTICITY

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REFLECTION AND REFRACTION OF PLANE PLASTIC WAVES

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§ 1. Let us consider an elastic-plastic medium 1, filling a half-space and bordering on an elastic medium 2. Since here we shall consider only plane one-dimensional motions, the properties of the elastic-plastic medium are sufficiently described by a diagram of uniaxial compression. Denoting the stress and strain by σ and ε, respectively, we represent the law of uniaxial compression in the form $\sigma = \tilde{\sigma} f(\varepsilon)$, $\tilde{\sigma} = \text{const}$ for the loading regime (Fig. 1). With respect to unloading, we assume that in the plastic region it occurs at constant deformation. We assume that on the compression diagram there is an initial rectilinear segment OA, corresponding to ideal elasticity; the continuation of the segment OA determines the point C on the diagram. The branch CD corresponds to the values of the stress in a shock wave propagating through the undisturbed medium. We further assume that on the branch CD the inequality $f(\varepsilon)/\varepsilon < f'(\varepsilon)$ holds, which is consistent with notions concerning real diagrams.

Considering the problem in Lagrangian coordinates h, t, we have

$$x(h,t) = h + u(h,t), \tag{1}$$

where u is the displacement and x is the Eulerian coordinate.

Fig. 1

We assume that the incident wave is caused by a shock load applied in the plane $h = 0$, monotonically decreasing from its initial value. The external load is described by the function $\sigma = \sigma_0(t)$, $\sigma_0(0) > \tilde{\sigma}$, $\sigma_0(t)$ is a decreasing function. For the coordinate $h_*(t)$ of the shock front of the incident wave, the following first-order differential equation holds:

$$h'_* \, \varphi(h'_*) \, h_* = \int_0^t \frac{\sigma_0(\tau)}{\rho_0} \, d\tau, \qquad h_*(0) = 0, \tag{2}$$

where the function $\varphi(h'_*) = \varepsilon$ is obtained as a result of solving the equation

$$h'^2_* = \frac{\tilde{\sigma}}{\rho_0} \frac{f(\varepsilon)}{\varepsilon}$$

with respect to ε. Through $h_*(t)$ all the remaining parameters of the incident wave are expressed. We note that the particle velocity $v(h,t)$ in the incident wave is a function only of t, so that $v(h,t) \equiv v(t)$.

As a result of the incidence of the wave on the plane of separation ($h = h_0$) of media 1 and 2, waves appear: a reflected wave (quantities referring to it are denoted by the subscript 1) and a refracted wave (subscript 2). The construction of the reflected wave under analogous conditions was considered in work ([1]). As in ([1]), the solution is constructed under the assumption that the reflected wave is more intense than the incident one. At the contact boundary of media 1 and 2, the conditions of continuity-

of the displacement vector and the stress components associated with the contact areas.

All quantities describing the act of reflection–refraction can be expressed in terms of two basic functions: $h_{1*}(t)$, the coordinate of the front of the reflected wave, and $V(t)$, the velocity of the boundary particles. In the reflected and refracted waves the particle velocities are functions only of time; hence $V(t) = v_1(t) = v_2(t)$. For the two indicated unknown functions there are two equations, (3) and (4):

$$\frac{f(\omega)}{f[\varepsilon(h_{1*})]} = 1 - \beta \frac{f[\varepsilon(h_*)]}{f[\varepsilon(h_{1*})]} \frac{h'_{1*}}{h'_*}, \qquad h_{1*}(0) = h_0,$$

$$\omega = \varepsilon(h_{1*}) \left(1 - \beta \frac{\varepsilon(h_*)}{\varepsilon(h_{1*})} \frac{h'_*}{h'_{1*}}\right), \qquad \beta = 1 - \frac{V(t)}{v(t)}; \tag{3}$$

$$\rho_0(h_{1*} - h_0)\dot{V}(t) + (\rho_0 h'_{1*} - \rho_2 a_2)V(t) = \rho_0 v h'_{1*} - \sigma_b(h_{1*}),$$

$$|V(0)| < +\infty. \tag{4}$$

Here σ_b is a function determined by the incident wave; a_2 is the velocity of the longitudinal wave in the second medium; ρ_2 is its density.

Equation (3) can be replaced by an approximate one obtained by expanding $f(\omega)$ in a series and retaining two terms; the latter equation is integrated in quadratures:

$$\int_{h_0}^{h_*} \frac{dh}{(f[\varepsilon(h)]/\varepsilon(h))^{1/2}} + \int_{h_0}^{h_{1*}} \frac{dh}{(f'[\varepsilon(h)])^{1/2}} = 0. \qquad (3')$$

Let us consider the particular case in which the incident wave is specified by a Heaviside function (has the form of a "step"). The reflected and refracted waves then have the same form; the velocities of the fronts of all waves are constant. Equation ($3'$) gives:

$$\left(\frac{a_1}{a}\right)^2 = \frac{f'(\varepsilon_0)}{f(\varepsilon_0)/\varepsilon_0}, \qquad (5)$$

where ε_0 is the strain in the incident wave.

The interpretation of this formula is of interest; as is seen from Fig. 1,

$$\left(\frac{a_1}{a}\right)^2 = \frac{\operatorname{tg}\alpha_1}{\operatorname{tg}\alpha}. \qquad (5')$$

The stress σ_1 in the reflected wave is

$$\sigma_1 = \frac{\rho_0 v\,(a + a_1)\rho_2 a_2}{\rho_2 a_2 + \rho_0 a_1}.$$

The coefficient K of reflection is

$$K = \frac{\sigma_{\text{refl}}}{\sigma_{\text{inc}}} = \frac{\sigma_1}{\rho_0 v a} = \frac{1 + a_1/a}{1 + \dfrac{\rho_0}{\rho_2}\dfrac{a_1}{a}\dfrac{a}{a_2}}. \qquad (6)$$

Formulas ($5'$) and (6) make it possible, from the properties of the compression diagram, to judge visually the influence of the intensity of the incident wave on the change in the reflection coefficient. As the intensity of the incident wave increases, K may increase, remain constant (for $f(\varepsilon) = \varepsilon^n$), or decrease; this depends on the geometric properties of the curve $\sigma = \sigma f(\varepsilon)$. For reflection to occur with an increase in intensity, the inequality $a_2\rho_2 > a\rho_0$* is necessary and sufficient.

§ 2. One may imagine a case in which the second medium is also elastoplastic (in the sense described above, $\sigma = \sigma_2 f_2(\varepsilon)$) and the refracted shock wave propagates through the undisturbed medium. In this case

* The results set forth above were reported by the authors at the Symposium on Elastoplastic Waves in Baku, 10 X 1964; see ([2]).

to the number of unknown functions there is added $h_{2*}(t)$, the coordinate of the front of the refracted wave. For the 3 unknown functions h_{1*}, h_{2*}, V we have 3 equations, one of which, the first, is the previous equation (3); to it are added the two others, (7) and (8):

$$[\rho_2(h_{2*} - h_0) - \rho_0(h_{1*} - h_0)]\dot{V}(t) + (\rho_2 h'_{2*} - \rho_0 h'_{1*})V(t) = \\ = \sigma_b(h_{1*}) - \rho_0 v h'_{1*}, \qquad |V(0)| < +\infty, \tag{7}$$

$$\frac{f_2(V/h'_{2*})}{V/h'_{2*}} = \frac{\rho_0}{\sigma_2} h'^2_{2*}. \tag{8}$$

In the case of an incident wave in the form of a step, the problem reduces to determining 3 constants a_1, a_2, V. The constant a_1 is determined by formula (5); for a_2 and V we have

$$V = \frac{\rho_0 v(a + a_1)}{a_2\rho_2 + a_1\rho_0}, \qquad \frac{f_2(V/a_2)}{V/a_2} = \frac{\rho_2}{\sigma_2} a_2^2.$$

It can be proved that the case considered is certainly realized if the graph of the function $f(\varepsilon)$ has a vertical asymptote and the incident wave is sufficiently intense.

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CITED LITERATURE

1. N. V. Zvolinskii, G. V. Rykov, *Prikl. matem. i mekh.*, **27**, no. 1 (1963).
2. N. V. Zvolinskii, G. V. Rykov, Abstracts of the All-Union Symposium on the Propagation of Elastic-Plastic Waves in Continuous Media, 7–14 X 1964, Baku, 1964, p. 12.

Note: Figure translations are in progress. See original paper for figures.