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**Abstract**

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# ROTATION OF THE PLANE OF POLARIZATION OF LIGHT BY CRYSTALS UNDER THE ACTION OF AN ELECTRIC FIELD

*(Presented by Academician A. V. Shubnikov on 8 VI 1964)*

Tetragonal crystals of the symmetry class $\bar{4} \cdot m$ of the ammonium dihydrogen phosphate (ADP) type possess a considerable linear electro-optic effect.

The optical indicatrix of these crystals is an ellipsoid of revolution. Under the action of a homogeneous electric field, whose intensity vector $\mathbf{E}$ is directed obliquely to the crystallophysical axes $x, y, z$, the main effect on this ellipsoid is exerted by the component $E_z$, while the action of the components $E_x$ and $E_y$ is so insignificant that in practice it may be neglected. Under the action of a varying field $E_z$, the ellipsoid of revolution is transformed into a triaxial ellipsoid varying in its shape, the directions of whose principal axes remain unchanged. In this case the magnitude of the principal axis $z$ of the ellipsoid does not change, while the other two principal axes, situated at an angle of 45° to the axes $x, y$, acquire increments equal in magnitude but opposite in sign, proportional to the field intensity $E_z$.

The ellipses of the section of the optical indicatrix by the planes (100) and (010) undergo no changes; in the planes (001), $(hk0)$, $(hhl)$, and $(h\bar{h}l)$ they change only their shape, without changing the directions of their axes, while in the planes $(0kl)$, $(h0l)$, and $(hkl)$ they change not only their shape but also the directions of their axes.

Of particular interest is the last case, when the ellipse rotates and at the same time changes its shape under the action of the field $E_z$, in particular, when the light is directed along the normal to the plane $(0kl)$ or $(h0l)$ of the ellipse, i.e., the direction of the light $z'$ in the crystal is parallel to one of the coordinate planes $yz$ or $xz$ and makes a small angle $\alpha$ with the $z$ axis. In this case the magnitude of the birefringence in the indicated direction of the light $z'$ in the crystal in the absence of a field depends on the angle $\alpha$ and on the magnitude of the birefringence of the crystal $\delta n$ in the direction transverse to the optical axis:

$$\delta n_{z'_0} = \delta n \cdot \sin^2 \alpha.$$

Fig. 1

Figure 1: Fig. 1

Under the action of the electric field $E_z$ on the crystal, the ellipse will rotate through a certain angle $\chi_{z'}$ in its plane and at the same time change its shape, i.e., the magnitude of the birefringence $\delta n_{z'}$ will become different. Omitting the derivation, we give the dependence of these quantities on the field intensity $E_z$, the electro-optic constant of the crystal $r_{63}$, the refractive index of the ordinary ray $n_0$, the magnitude of the maximum birefringence of the crystal $\delta n$, and the angle $\alpha$:

$$\operatorname{tg} 2\chi_{z'} = \frac{n_0^3 r_{63} E_z}{\delta n \cdot \sin^2\alpha}; \qquad \delta n_{z'} = \sqrt{(\delta n \cdot \sin^2\alpha)^2 + \left(n_0^3 r_{63} E_z\right)^2}.$$

These dependences can be interpreted geometrically by a right triangle (Fig. 1), from which, taking into account the smallness of the quantity $\delta n_E = n_0^3 r_{63} E_z$ in comparison with $\delta n_{z'}^0 = \delta n \sin^2\alpha$, the increment of the birefringence $\Delta(\delta n_{z'}) = \delta n_{z'} - \delta n_{z'}^0$ is easily determined:

$$\Delta(\delta n_{z'}) \simeq \frac{1}{2}\delta n_E \cdot \operatorname{tg} 2\chi_{z'} = \frac{\delta n_E^2}{2\delta n_z^0} = \frac{(n_0^3 r_{63} E_z)^2}{2\delta n \sin^2\alpha}.$$

If the direction of oscillations of the incident linearly polarized light and its direction in the crystal are parallel to the coordinate plane $xz$ and $yz$, then in the absence of a field this direction of oscillations will not change after the passage of light through the crystal, since it is parallel to the principal axis of the ellipse of the section of the indicatrix by the plane normal to the path of the light. Under the action of the field $E_z$, however, as a result of rotation through the angle $\chi_{z'}$ and simultaneous change in the shape of this ellipse, i.e., the appearance of an increment of the birefringence $\Delta(\delta n_{z'})$, the light emerging from the crystal will no longer be linearly polarized, but will become elliptically polarized, with ellipticity of the polarization ellipse $a_2/a_1 = \operatorname{tg}\gamma$ and with angle of rotation $\beta$ of its axis $a_1$ relative to the direction of the principal axis of the rotated ellipse of the indicatrix section.

**Fig. 1**

Omitting the derivation, we give the formulas expressing the dependence of the parameters of elliptically polarized light $\gamma$ and $\beta$ on the angle $\alpha$ and the field strength $E_z$:

$$\begin{aligned} \sin 2\gamma &= \sin 2\chi_{z'} \cdot \sin\left\{\left[\delta n_{z'}^0 + \Delta(\delta n_{z'})\right]\frac{2\pi d_{z'}}{\lambda}\right\}, \\ \operatorname{tg} 2\beta &= \operatorname{tg} 2\chi_{z'} \cdot \cos\left\{\left[\delta n_{z'}^0 + \Delta(\delta n_{z'})\right]\frac{2\pi d_{z'}}{\lambda}\right\}, \end{aligned} \tag{1}$$

where $d_{z'}$ is the path length of light in the crystal along the direction $z'$, and $\lambda$ is the wavelength of light.

Of special interest is the case when the path difference of the rays in the crystal along the direction $z'$ in the absence of a field is equal to a half-wave

$$\Gamma^0_{z'} = d_{z'} \cdot \delta n^0_{z'} = \lambda/2,$$

and under the action of the field $E_z$ it acquires a small increment

$$\Delta\Gamma_{z'} = d_{z'} \cdot \Delta(\delta n_{z'}) \ll \lambda/2.$$

In this case, after the corresponding substitution, formulas (1) take the form

$$\sin 2\gamma = -\sin 2\chi_{z'} \cdot \sin\left(\frac{\pi}{2}\operatorname{tg}^2 2\chi_{z'}\right), \qquad \gamma \simeq +2\pi \cdot \chi^3_{z'};$$

$$\operatorname{tg} 2\beta = -\operatorname{tg} 2\chi_{z'} \cdot \cos\left(\frac{\pi}{2}\operatorname{tg}^2 2\chi_{z'}\right), \qquad \beta \simeq -\chi_{z'};$$

where

$$\operatorname{tg} 2\chi_{z'} = \frac{\Delta\Gamma_{z'}}{\left(\frac{\lambda}{2}\right)} = \frac{2d_{z'} n_0^3 r_{63} E_z}{\lambda}.$$

Substituting the numerical values of the quantities for an ADP crystal: $n_0 = 1.5254$, $n_e = 1.4798$, $\delta n = 0.0456$, $r_{63} = 2.54 \cdot 10^{-7}$ CGSE and the field strength $E_z = 10$ kV/cm, with the thickness of the $Z$-cut plate $d_z = 0.2$ cm and $\lambda = 5 \cdot 10^{-5}$ cm, we obtain: $\alpha = 3^\circ$; $\chi_{z'} = 7^\circ$; $\gamma = 0.0044 \simeq 0.5\%$; $\beta = 7^\circ$; $\varphi = \chi_{z'} - \beta = 14^\circ$.

Under the action of the electric field $E_z$, at the exit from the crystal we obtain practically linearly polarized light ($\gamma = 0.5\%$), whose plane of polarization is rotated by an angle $\varphi = 14^\circ$ relative to its initial position in the absence of the field. Using the formulas given above, the dependence has been calculated of the angle of rotation of the plane of polarization $\varphi$ on the voltage $V_z$ at the electrodes of a $z$-cut plate of an ADP crystal and on the resulting optical path difference of the rays $\Delta\Gamma_{z'}$ along the $z'$ axis (Fig. 2), as well as the dependence of the ellipticity $a_2/a_1 = \operatorname{tg}\gamma$ on the angle of rotation $\varphi$ of the plane of polarization for the case when the direction of oscillations of the incident linearly polarized light is parallel to one of the axes of the ellipse of the section of the indicatrix by the plane normal to the ray.

**Fig. 2.** Linear dependence of the angle $\varphi$ of rotation of the plane of polarization of light by a $z$-cut plate of an ADP crystal on the voltage $V_z$ at its transparent electrodes, and the corresponding dependences of the ellipticity $\operatorname{tg}\gamma$ of the light

Fig. 2

Figure 2: Fig. 2

oscillations on the angle of rotation of the plane of polarization $\varphi$, for the direction of light in the plate parallel to the plane $xz$ or $yz$, at which the optical path difference of the rays in the crystal is $\lambda/2$. $\operatorname{tg}\gamma$—the direction of oscillations of the incident light coincides with one of the directions of oscillations in the crystal; $\operatorname{tg}\gamma_{\min}$—as a result of the rotation, the plane of polarization of the transmitted light coincides with the plane of polarization of the incident light; $\operatorname{tg}\gamma_{\max}$—as a result of rotation through an arbitrary angle, the plane of polarization of the transmitted light makes an angle of 45° with the directions of oscillations in the crystal.

Also shown here is the dependence of the minimum possible ellipticity $\operatorname{tg}\gamma_{\min}$, when the direction of oscillations of the incident light is deflected by the prescribed maximum angle of rotation of the plane of polarization relative to one of the axes of the ellipse of the section of the indicatrix and, under the action of a field of one sign, rotates until it coincides with the direction of the corresponding axis of the ellipse. If, however, the direction of oscillations of the incident light makes such an angle with the direction of the axes of the ellipse of the section of the indicatrix that, after rotation by the prescribed maximum angle, the plane of polarization of the light assumes a position at an angle of 45° to the axes of the ellipse of the section of the indicatrix, the ellipticity of the oscillations will be maximal ($\operatorname{tg}\gamma_{\max}$).

In addition to the considered directions $\langle 0kl\rangle$ and $\langle h0l\rangle$ of light in ADP-type crystals, the effect of rotation of the plane of polarization may also occur for all oblique directions $\langle hkl\rangle$. But for the latter directions the effect will be weaker and, as they approach the directions $\langle 100\rangle$, $\langle 010\rangle$, $\langle 001\rangle$, $\langle hk0\rangle$, $\langle hhl\rangle$, and $\langle h\bar{h}l\rangle$, it will decrease to zero. Therefore we omit the analysis of this effect for the general case of an oblique direction of light $\langle hkl\rangle$.

In conclusion, the following conclusions may be drawn.

It has been established that in tetragonal crystals of symmetry class $\bar{4}\cdot m$ there may occur the phenomenon of rotation of the plane of polarization of light under the action of an electric field. This phenomenon is a limiting special case of the electro-optic effect, when light propagates parallel to the coordinate planes $xz$ and $yz$, while the electric field is directed along the $z$ axis. Under this condition, the condition for the realization of the rotation effect along some of the indicated directions is that the path difference of the rays in the crystal in the absence of the field be equal to one half-wave or to an odd number of half-waves.

The rotation effect is accompanied by a small ellipticity of the oscillations, increasing as the angle of rotation of the plane of polarization increases; for example, when the plane of polarization is rotated by an angle up to $\pm 13°$, the ellipticity increases to 1%, and in a whole series of practical applications such small ellipticity may be neglected.

An analogous phenomenon takes place in crystals of the ADP type when the directions of the light and of the electric field are along the $x$ (or $y$) axis, since in this case the ellipse of the section of the optical indicatrix by the $yz$ (or $xz$) plane likewise rotates, practically without changing its shape. However, the angle of rotation of the ellipse is so small (of the order of several angular minutes at reasonable electric-field strengths $E'_x$) that this phenomenon can be of only fundamental, but not practical, interest in comparison with the phenomenon considered above.

It may be asserted that the phenomenon described is characteristic of all crystals possessing an electro-optic effect and having such elliptical sections of the optical indicatrix which, under the action of an appropriately directed electric field, rotate in their own plane without substantially, or better still without at all, changing their shape. The condition for the realization of such a phenomenon will be that the path difference of the rays in the crystal along the direction of the normal to such a section of the indicatrix be equal to one half-wave or to an odd number of half-waves in the absence of the field.

It may also be asserted that in all cases where, in a homogeneous transparent bounded medium with natural or artificially produced birefringence along some direction, there is a path difference of the rays equal to one or to an odd number of half-waves, and, under the action either of an electric or magnetic field, or of mechanical stress, etc., the elliptical section of the optical indicatrix normal to the given direction of light rotates without changing its shape, there will occur a "pure" rotation of the plane of polarization of the light propagating in that direction. If, however, the rotation of the ellipse of the indicatrix section is accompanied by an increment of the path difference that is small in comparison with a half-wave, the rotation will no longer be "pure," since it will be accompanied by the appearance of a small ellipticity of the oscillations, increasing as the angle of rotation of the plane of polarization increases.

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*Note: Figure translations are in progress. See original paper for figures.*