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1965

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Abstract

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MASS FORMULAS IN THE SUPERMULTIPLET MODEL

(Presented by Academician N. N. Bogolyubov, January 12, 1965)

1. Recently, Gürsey, Radicati, and Pais proposed a model of strongly interacting particles ([1,2]), based on the group $SU(6)$. This model systematizes hadrons according to F-spin–spin supermultiplets. In papers ([3–5]), mass formulas were found for meson, baryon, and selected supermultiplets.

Fig. 1

In the present paper we shall give a formulation of this model in the language of quarks ([6]). The advantage of such an approach is that it makes it possible to use diagrammatic techniques for obtaining mass formulas.

2. Let us introduce into consideration a basic sextet of quarks $(q_{11}, q_{12}, q_{21}, q_{22}, q_{31}, q_{32})$ and represent it in the form

$$q_{ij} = u_i v_j, \tag{1}$$

where (u_1, u_2, u_3) is a unitary triplet, and (v_1, v_2) is a spin doublet. Symbolically, relation (1) may be written as follows: $6 = (3, 2)$. The quantum numbers of the particles u_i and v_j are given in Table 1 (B is baryon number, I is isospin, Y is hypercharge, J is spin).

Table 1

	B	I	Y	J
u_1	1/3	1/2	1/3	0
u_2	1/3	1/2	1/3	0
u_3	1/3	0	$-2/3$	0
v_1	0	0	0	1/2
v_2	0	0	0	1/2

Table 2

3	$v_1 v_1$		2	$(\{v_1 v_2\} v_1 - 2 v_1 v_1 v_2)/\sqrt{6}$

3	$\{v_1 v_2\}/\sqrt{2}$	2	$(2v_2 v_2 v_1 - \{v_1 v_2\} v_2)/\sqrt{6}$
3	$v_2 v_2$	4	$v_1 v_1 v_1$
1	$[v_1 v_2]/\sqrt{2}$	4	$(\{v_1 v_2\} v_1 + v_1 v_1 v_2)/\sqrt{3}$
2	$[v_1 v_2] v_1/\sqrt{2}$	4	$(v_2 v_2 v_1 + \{v_1 v_2\} v_2)/\sqrt{3}$
2	$[v_1 v_2] v_2/\sqrt{2}$	4	$v_2 v_2 v_2$

Table 3

8	$\bar{u}_2 u_1 (\bar{u}_2 u_1 - \bar{u}_2 u_2)/\sqrt{2} \bar{u}_1 u_2 \bar{u}_3 u_1 \bar{u}_3 u_2 \bar{u}_2 u_3$ [illegible] $\bar{u}_2 u_2 - 2\bar{u}_3 u_3)/\sqrt{6}$	8	$[u_3 u_1]\, u_1/\sqrt{2}$ [illegible]	10	$[u_2 u_3]\, u_1 -$ [illegible] $u_2 u_3]\, u_2/\sqrt{2} [u_1 u_2]\, u_1/\sqrt{2} [u_1 u_2]\, u_2/\sqrt{2} [u_3 u_1]\, u_3/\sqrt{2} [u_2 u_3]$	$u_1 u_1 u_1 \{u_1 u_1 u_2\}/\sqrt{3} \{u_1 u_2 u_2\}/\sqrt{3} u_2 u_2 u_2 \{$
1	$(\bar{u}_1 u_1 + \bar{u}_2 u_2 + \bar{u}_3 u_3)/\sqrt{3}$	1	$[u_1 u_2\, u_3]/\sqrt{6}$			

Following the standard method (7), we construct from quarks the supermultiplets of higher dimensionality:

$$\bar{6} \times 6 = 35 + 1 \quad \text{(mesons)},$$

$$6 \times 6 \times 6 = 70 + 70 + 56 + 20 \quad \text{(baryons and isobars)},$$

where

$$35 = (8,3) + (8,1) + (1,3) + (8,2) + (1,2),$$

$$1 = (1,1),$$

$$70 = (10,2) + (8,4) + (10,2),$$

$$56 = (10,4) + (8,2), \qquad 20 = (8,2) + (1,4).$$

The F-spin–spin multiplets (n,m) are given in Tables 2 and 3 (for an explanation of the notation $\{ab\}$, $[ab]$, $\{aab\}$, $\{abc\}$, $[abc]$ see Ref. (8)).

Fig. 2

3. Using the data of Tables 2 and 3, it is not difficult to construct self-energy diagrams for mesons, baryons, and isobars, taking into account the breaking of the $S\tilde{U}$ (6)-symmetry in first order in the interaction

$$L = \bar{q}_{31}q_{31} + \bar{q}_{32}q_{32}.$$

For the 35-plet and 56-plet these diagrams are shown in Figs. 1 and 2 (wavy lines denote quarks q_{11}, q_{12}, q_{21}, q_{22},

straight lines are quarks q_{31}, q_{32}, black vertices are the interaction L.

The diagrams presented immediately lead to the mass formulas.

35-plet (the particle symbols denote the squares of their masses):

$$\omega = \rho, \qquad \varphi + \rho = 2K^*; \qquad K = \frac{1}{4}(3\eta + \pi); \qquad K^* - \rho = K - \pi.$$

56-plet (the particle symbols denote their masses):

$$\Omega - \Xi^* = \Xi^* - Y_1^* = Y_1^* - N^*;$$

$$\Xi - \Sigma = \Sigma - N, \qquad \Lambda = \Sigma; \qquad \Xi^* - Y_1^* = \Xi - \Sigma.$$

4. In an analogous way, the mass formulas for the 70-plet and the 20-plet are obtained.

70-plet:

$$\Omega_0^{1/2} - \Xi_{1/2}^{*1/2} = \Xi_{1/2}^{*1/2} - Y_1^{*1/2} = Y_1^{*1/2} - N_{3/2}^{*1/2};$$

$$\Xi_{1/2}^{3/2} - Y_1^{3/2} = Y_1^{3/2} - N_{1/2}^{3/2}, \qquad Y_0^{3/2} = Y_1^{3/2};$$

$$\Xi_{1/2}^{1/2} - Y_1^{1/2} = Y_1^{1/2} - N_{1/2}^{1/2}, \qquad Y_0^{1/2} + Y_0^* = 2Y_1^{1/2}, \qquad Y_0^{1/2}Y_0^* = \frac{2}{3}\,Y_1^{1/2}Y_1^{1/2};$$

$$\Xi_{1/2}^{*1/2} - Y_1^{*1/2} = \Xi_{1/2}^{3/2} - Y_1^{3/2} = \Xi_{1/2}^{1/2} - Y_1^{1/2}.$$

20-plet:

$$\Xi_{1/2}^{\prime 1/2} - Y_1^{\prime 1/2} = Y_1^{\prime 1/2} - N_{1/2}^{\prime 1/2}, \qquad Y_0^{\prime 1/2} = Y_1^{\prime 1/2}.$$

The formulas we have found for the 70-plet differ from the formulas given in works ([3],[5]).

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Received
7 I 1965

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Note: Figure translations are in progress. See original paper for figures.