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Abstract

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GEOPHYSICS

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THE DEPENDENCE OF THE WIND COEFFICIENTS IN THE ARCTIC BASIN ON WIND VELOCITY AND ICE THICKNESS

(Presented by Academician L. I. Sedov, 3 III 1965)

Earlier, expressions were obtained for the components of the velocity of purely wind-driven ice drift ([1,2])

$$U = \frac{1}{2\rho a A}\frac{T_x + (1+2m)T_y}{1+2m+2m^2};$$

$$V = \frac{1}{2\rho a A}\frac{-(1+2m)T_x + T_y}{1+2m+2m^2}, \tag{1}$$

in which T_x, T_y are the components of the tangential wind stress acting on the ice; ρ is the density of seawater; A is the kinematic coefficient of vertical exchange in the hydrosphere; a and m are quantities related to the exchange coefficient A, the Coriolis parameter Ω, the density ρ_l, and the ice thickness h by the relations

$$a = \sqrt{\frac{\Omega}{2A}}; \tag{2}$$

$$m = \frac{\rho_l}{\rho} a h. \tag{3}$$

From expressions (1) one can obtain the following formula for the magnitude of the velocity of purely wind-driven ice drift v:

$$v = \frac{T}{\sqrt{2\rho a A}}\frac{1}{\sqrt{1+2m+2m^2}}. \tag{4}$$

Introducing the depth of friction in seawater beneath the ice,

$$D = \pi/a, \tag{5}$$

and using relation (2), we obtain

$$v = \frac{\sqrt{2\pi}T}{\rho\Omega D} \cdot \frac{1}{\sqrt{1 + 2\pi\frac{\rho_l}{\rho}\frac{h}{D} + 2\pi^2\left(\frac{\rho_l}{\rho}\right)^2\left(\frac{h}{D}\right)^2}}. \tag{6}$$

We relate the magnitude of the tangential wind stress T, entering formula (6), to the magnitude of the wind velocity W by the quadratic dependence

$$T = \gamma W^2, \tag{7}$$

where γ is a constant coefficient whose value, in the absence of other data, may be taken equal to its value in the case of an ice-free sea ([2]).

The friction depth D entering formula (6) will be determined by an indirect method, using considerations of dimensional theory. We shall assume that the friction depth D depends on the velocity of purely wind-driven ice drift v and on the Coriolis parameter Ω. From dimensional considerations we obtain

$$D = Cv/\Omega, \tag{8}$$

where c is a constant dimensionless quantity.

Substituting (7) and (8) into formula (6). Introducing here the wind coefficient k, defined as the ratio of the speed of the purely wind-induced drift of the ice to the speed of the wind causing this drift:

$$k = v/w, \tag{9}$$

we obtain

$$k^2 = \frac{k_0^2}{\sqrt{1 + \eta\frac{k_0^2}{k}\frac{h}{w} + \frac{\eta^2}{2}\frac{k_0^4}{k^2}\left(\frac{h}{w}\right)^2}}, \tag{10}$$

where k_0 is the value of the wind coefficient in the limiting case when the ice thickness tends to zero ($k_0 = k_{h\to 0}$), and

$$\eta = \sqrt{2}\rho_\ell\Omega/\gamma. \tag{11}$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Fig. 1

Fig. 2. a — "G. Sedov" ; b — "North Pole-1" ; v — "North Pole-2"

It is more convenient to solve relation (10) not with respect to the wind coefficient k, but with respect to the quantity h/w. For h/w we obtain the equation

$$\left(\frac{h}{w}\right)^2 + 2\frac{k}{\eta k_0^2}\frac{h}{w} + 2\frac{k^4 - k_0^4}{\eta^2 k_0^4 k^2} = 0, \tag{12}$$

whose solution has the form

$$\frac{h}{w} = \frac{1}{\eta k}\left[\sqrt{2 - \left(\frac{k}{k_0}\right)^4} - \left(\frac{k}{k_0}\right)^2\right]. \tag{13}$$

To determine the coefficient k_0, we transform (10) to the form

$$\left(\frac{1}{k_0^2}\right)^2 + \frac{\eta}{k}\frac{h}{w}\frac{1}{k_0^2} + \frac{\eta^2}{2k^2}\left(\frac{h}{w}\right)^2 - \frac{1}{k^4} = 0 \tag{14}$$

and solve equation (14) with respect to k_0. We obtain

$$k_0 = \frac{1}{\sqrt{\sqrt{\frac{1}{k^4} - \left(\frac{\eta}{2k}\frac{h}{w}\right)^2} - \frac{\eta}{2k}\frac{h}{w}}}. \tag{15}$$

Suppose that, from observations of ice drift, for ice whose thickness is equal to $\bar{h}$, the value of the wind coefficient $\bar{k}$ is known at some wind speed $\bar{w}$. From these data, using formula (15), one can compute the required coefficient k_0. Then, after determining the value of η (formula (11)), one can use formula (13) to calculate the values of the ratio of ice thickness to wind speed h/w for various values of the wind coefficient k, and construct a graph of the dependence between k and h/w, which makes it possible to determine the wind coefficient k, and then also the speed of the purely wind-induced drift $v = kw$, as a function of the wind speed w and the ice thickness h.

As examples, Fig. 1 presents graphs of the dependence of the wind coefficient k on the ratio of ice thickness to wind speed

h/w in the following four cases: 1–$\bar{h}/\bar{w} = 0.5$ for $\bar{k} = 0.015$; 2–$\bar{h}/\bar{w} = 0.5$ for $\bar{k} = 0.017$; 3–$\bar{h}/\bar{w} = 1$ for $\bar{k} = 0.015$; 4–$\bar{h}/\bar{w} = 1$ for $\bar{k} = 0.017$. In these cases, calculations by formula (15) give values of k_0 equal, respectively, to 0.0171; 0.0199; 0.0217 and 0.0271. In the calculations the adopted values were $\Omega = 1.45 \cdot 10^{-4}$ sec^{-1} (84° north latitude), $\rho_l = 0.9$ g/cm^3 and $\gamma = 3.25 \cdot 10^{-6}$ g/cm^3, for which $\eta = 56.78$ sec^{-1}. Figure 2 gives graphs of the dependence of the wind coefficient k on the wind speed w, varying from 3 to 10 m/sec, for an ice thickness $h = 3$ m in these four cases.

Table 1

w, m/sec	"G. Sedov" number of resulting points	"G. Sedov" w_{av}	"G. Sedov" k_{av}	"North Pole-1" number of resulting points	"North Pole-1" w_{av}	"North Pole-1" k_{av}	"North Pole-2" number of resulting points	"North Pole-2" w_{av}	"North Pole-2" k_{av}
From 3 to 4	93	3.5	0.0150	26	3.6	0.0146	45	3.5	0.0196
» 4 » 5	102	4.4	0.0164	23	4.5	0.0152	43	4.5	0.0219
» 5 » 6	78	5.5	0.0159	16	5.4	0.0140	21	5.5	0.0197
» 6 » 7	44	6.5	0.0177	8	6.6	0.0168	10	6.3	0.0201
» 7 » 8	38	7.5	0.0177	6	7.5	0.0147	14	7.5	0.0219
» 8 » 9	19	8.5	0.0179	3	8.7	0.0162	3	8.4	0.0217
» 9 » 10	11	9.5	0.0188	2	9.3	0.0211	2	9.1	0.0138

According to the results obtained, with a decrease in ice thickness or an increase in wind speed the wind coefficient increases; moreover, as is seen from the examples considered, this increase can be quite substantial (when h/w decreases from 2 to 1 sec, the coefficient k increases by approximately a factor of two).

We note that the conclusion that the wind coefficient grows with increasing wind speed is confirmed by analysis of actual observations of ice drift in the

Arctic Basin. As an example, Table 1 gives mean values of the wind coefficient obtained on the basis of processing observations of the drift of the "G. Sedov" and the stations "North Pole-1" and "North Pole-2" according to resultants between astronomical points. The corresponding points are plotted in Fig. 2.

As is seen from Fig. 2, the agreement between the results of the calculations and the observations is good.

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Note: Figure translations are in progress. See original paper for figures.