PHYSICS

1965

SovietRxiv

Abstract

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PHYSICS

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GENERATION OF INDUCED RADIATION BY A CRYSTAL MOVING INSIDE A RESONATOR

(Presented by Academician I. V. Obreimov on 31 X 1964)

As is known, in a uniformly pumped crystal, during the generation of induced radiation in the direction of the resonator axis, inhomogeneities of the inverse population appear along the length of the crystal, with linear dimensions of the order of the radiation wavelength ([1]). In the preceding work ([2]) it was shown that, in any case at room temperature, diffusion of the inverse population is unlikely to eliminate these inhomogeneities even in the stationary regime. Therefore the problem of generation by a crystal moving relative to the resonator naturally arises.

As the basis of the consideration we shall take a model of axial modes arising in a resonator with plane mirrors at the ends of the crystal. In contrast to work ([1]), we shall assume that the active centers move relative to the mirrors with velocity v along the resonator axis.

The system of kinetic equations for the inverse population $n(z,t)$ and the numbers of photons $N_i(t)$ in the i-th axial mode has the form:

$$\frac{dn}{dt} = -\frac{n-n_0}{\tau} - \sum_i D g_i n P_i,$$

$$\frac{dN_i}{dt} = -\gamma_i N_i + \int_0^L D g_i n P_i \, dz, \qquad (1)$$

where τ is the spontaneous decay time; L is the length of the resonator; γ_i is the relative number of photon losses in the i-th mode per unit time; n_0 is the stationary inverse population in the absence of induced radiation; g_i is the ordinate of the normalized luminescence line curve at the frequency of the i-th resonator mode; D is a quantity proportional to the Einstein coefficient; $P_i = N_i(t)(1-\cos 2\pi m_i z/L)$ is the energy density of the i-th mode at the point z.

The derivative dn/dt, in the case of motion of the active centers with velocity v along the system of standing axial waves, may be written in the form

$$dn/dt = \partial n/\partial t + v\,\partial n/\partial z. \tag{2}$$

We shall seek a stationary solution of system (1), putting $\partial n/\partial t$ and dN_i/dt equal to zero. Then we have

$$v\frac{dn}{dz} = -\frac{n-n_0}{\tau} - \sum_i Dg_i nN_i\left(1-\cos\frac{2\pi m_i}{L}z\right); \tag{3a}$$

$$0 = -\gamma_i N_i + \int_0^L Dg_i nN_i\left(1-\cos\frac{2\pi m_i}{L}z\right)\,dz. \tag{3b}$$

Equation (3a) has the solution:

$$n(z) = \exp\left\{-\frac{z}{\tau v} - \frac{D}{v}\sum_i g_{iN}i\left(z-\frac{L}{2\pi m_i}\sin\frac{2\pi m_i}{L}z\right)\right\}\times$$

$$\times\left[C_1 + \frac{n_0}{\tau v}\int\exp\left\{\frac{z}{\tau v} + \frac{D}{v}\sum_i g_{iN}i\left(z-\frac{L}{2\pi m_i}\sin\frac{2\pi m_i}{L}z\right)\right\}dz\right]. \tag{4}$$

To determine C_1 we use the boundary condition $n(0) = n(L)$. Then, in the approximation $D\tau\sum_i g_iN_i \ll 1$ (1), we obtain $C_1 = 0$ and

$$n(z) = n_0\left[1 + D\tau\sum_i g_iN_i\chi_i\sin\frac{2\pi m_i}{L}z - D\tau\sum_i g_iN_i - D\tau\sum_i g_iN_i\frac{\chi_i\sin 2\pi m_i z/L - \cos 2\pi m_i z/L}{1+1/\chi_i^2}\right], \tag{5}$$

where $\chi_i = L/2\pi m_i\tau v = \lambda_i/4\pi\tau v$.

In the case of ruby, $\chi \approx 2\cdot 10^{-3}\,(\text{cm/sec})\cdot v^{-1}$, whence it is seen that already at velocities $v \sim 10^{-2}$ cm/sec, $\chi \ll 1$. Taking this into account, we have:

$$n(z) = n_0\left[1 - D\tau\sum_i g_iN_i + D\tau\sum_i g_iN_i\chi_i\sin\frac{2\pi m_i}{L}z + D\tau\sum_i g_iN_i\chi_i^2\cos\frac{2\pi m_i}{L}z\right]. \tag{6}$$

We next substitute (6) into (3b) and, under the condition $N_i \neq 0$, obtain a system of linear algebraic equations for determining the number of photons N_i in the i-th generated mode:

$$-\frac{\gamma_i}{Dg_i n_0 L} + 1 - D\tau \sum_k g_k N_k - \frac{D\tau}{2} g_i N_i \chi_i^2 = 0. \tag{7}$$

If $2j+1$ modes are generated and mode $j+1$ coincides with the center of the luminescence line, then, assuming the luminescence line shape to be Lorentzian with half-width $2\Delta\nu$, we have:

$$g_i = g/[1+\beta(j-i+1)^2], \tag{8}$$

where $g = (\pi \cdot \Delta\nu)^{-1}$, $\beta = (\delta\nu/\Delta\nu)^2$, and $\delta\nu = c/2L$ is the frequency difference of neighboring axial modes. Summing the equations of system (7), we find:

$$\sum_{i=1}^{2j+1} g_i N_i = \frac{2j+1-(1/\alpha)\left[2j+1+\frac{1}{3}\beta j(j+1)(2j+1)\right]}{D\tau(2j+1+\chi^2/2)}; \tag{9}$$

here we have put $\chi_i = \chi$, $\gamma_i = \gamma$, and introduced the notation $\alpha = Dgn_0L/\gamma$. Substituting (9) into (7), we obtain

$$N_i = \frac{2\left[\frac{1}{2}\chi^2(\alpha-1)+\beta^{1/3}j(j+1)(2j+1)-\beta(2j+1+\frac{1}{2}\chi^2)(j-i+1)^2\right]\left[1+\beta(j-i+1)^2\right]}{D\tau g\alpha\chi^2(2j+1+\frac{1}{2}\chi^2)}. \tag{10}$$

From expression (10) there follows the condition for determining the number of generated modes as a function of the parameters α, β, and χ, namely, putting $N_{2j+2} = 0$, we have:

$$\chi^2(\alpha-1)/2\beta - (2j+1+\chi^2/2)(j+1)^2 + \tfrac{1}{3}j(j+1)(2j+1) = 0. \tag{11}$$

Thus, relations (10) and (11) contain the result of the influence of the translational motion of the inverse population on the radiation power in individual modes and on the number of modes that have reached generation. In particular, from (11) it is seen that, at the corresponding velocities, in the stationary regime one can achieve generation of only one mode. Indeed, putting $j = 0$, we obtain:

$$\chi = \sqrt{2\beta/(\alpha-1-\beta)}. \tag{12}$$

In the particular case of ruby, for $\tau = 3 \cdot 10^{-3}$ sec, $\alpha = 2$, and $\beta = 10^{-5}$, on the basis of (12) it follows that for $v > 0.5$ cm/sec in the stationary regime generation will occur in one mode.

On the other hand, from (10) it is easy to obtain a formula for the total number of generated photons:

$$\sum_{i=1}^{2j+1} N_i = \frac{2}{\alpha D\tau g\chi^2}\left\{\left[\frac{\alpha-1}{2}\chi^2+\beta-\frac{j(j+1)(2j+1)}{3}\right]\times\right.$$

$$\left.\times\left[2j+1+\beta\frac{j(j+1)(2j+1)}{3}\right]\frac{1}{2j+1+\chi^2/2}-\beta\frac{j(j+1)(2j+1)}{3}-\frac{\beta}{15}j(j+1)(2j+1)(3j^2+3j-1)\right\}. \quad (13)$$

It is difficult to analyze this expression, since j, determined from (11), depends on α and χ, and when these parameters change it runs through integer values. For those values of α and χ for which equation (11) has integer solutions, formula (13) takes the simpler form:

$$\sum N = \frac{\alpha-1-\beta(j+1)^2}{\alpha D\tau g}\left[1+\beta\frac{j(j+2)}{5}\right]. \quad (14)$$

Let us consider two cases—that of a large and of a small number of modes.

Thus, when $\beta = 10^{-5}$ and $\alpha = 2$, the number of modes generated by a stationary crystal is $2j+1 = 67$, and the total number of generated quanta practically does not change when the crystal moves. Physically this is understandable, since with a large number of modes the depletion of the inverse population becomes almost the same for all z, and therefore the motion of the crystal, while helping to reduce the number of generated modes, cannot lead to an increase in the number of photons.

If, on the contrary, even with a stationary crystal a single-mode regime is realized (for example $\alpha = 1.1$; $\beta = 5 \cdot 10^{-3}$), then the number of generated photons will depend substantially on the velocity of the crystal motion.

In this case (13) for $j = 0$ gives

$$\sum_{i=1}^{2j+1} N_i = N_1 = (\alpha-1)/D\tau\alpha(1+\chi^2/2), \quad (15)$$

whence it is seen that N_1 changes by 50% in going from a stationary crystal to one moving with a velocity corresponding to $\chi \approx 0$.

The model adopted in this paper of a moving medium with fixed mirrors at its end faces was chosen by us to simplify the mathematical calculations. Consideration of a resonator model with external mirrors, where the distance between the crystal end faces and the resonator mirrors was assumed small compared with the length of the crystal, gives results differing from those set forth only in that in the numerator of the expression for α, instead of the resonator length L, there stands the crystal length l.

The treatment carried out applies to active centers with a homogeneously broadened luminescence line.

The results obtained require experimental verification. The paper that has recently appeared[3], because of its brevity, does not allow us to make any comparisons.

The authors express their gratitude to Academician I. V. Obreimov for his attention to this work.

Note added in proof. Recently we have obtained experimental evidence of the phenomenon of narrowing of the spectrum of induced radiation when a crystal moves inside a resonator.

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Received
28 X 1964

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Note: Figure translations are in progress. See original paper for figures.