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Abstract

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ON A MECHANICAL ANALOGY OF A GYROSCOPIC STABILIZER IN THE PRESENCE OF ELASTIC COMPLIANCE OF ITS ELEMENTS

The equations of a gyroscopic stabilizer ([1]), compiled under the assumption of absolute rigidity of its elements, lead to incorrect values of the natural frequencies of oscillation of this electromechanical system and, in a number of cases, to erroneous conclusions about its stability. Taking into account the elastic compliance of the gyroscope suspension and of the mechanical transmissions of the gyrostabilizer, although it considerably complicates the investigation, nevertheless in most cases proves absolutely necessary.

1°. In essence, the mechanical system of a gyroscopic stabilizer (Fig. 1) is a system with distributed parameters. If it is considered to be free of friction, then, with the electrical circuits of the motor open, it must be regarded as a conservative system with an infinite discrete spectrum of frequencies.

However, the elastic compliance of the gyroscope-rotor bearings in the radial direction and the crushing of the contacting teeth of the reduction gear considerably exceed the elastic compliance of the remaining elements of the stabilizer. Therefore, in determining the lower frequencies of small oscillations of such a conservative system, it may be considered as if consisting of four absolutely rigid bodies: the gyroscope rotor, its casing, the outer suspension ring (together with the body being stabilized and the driven gear wheel of the reducer), and the motor rotor (together with the driving wheel). The first two bodies are elastically connected with one another; the hinge connecting the gyroscope casing with the outer suspension ring may be taken as nondeformable, and the mass of the casing itself may be neglected; the connection of the outer ring with the motor rotor is also elastic. Accordingly, the equations of small oscillations of such an idealized conservative system, located on a fixed base, may be represented in the form

$$A\frac{d^2\alpha}{dt^2} + H\frac{d\beta}{dt} = K(\psi - \alpha), \qquad A\frac{d^2\beta}{dt^2} - H\frac{d\alpha}{dt} = 0,$$

$$\Psi\frac{d^2\psi}{dt^2} = K(\alpha - \psi) + N(\theta - \psi), \qquad \Theta\frac{d^2\theta}{dt^2} = N(\psi - \theta). \tag{1}$$

Fig. 1. Schematic of a gyroscopic stabilizer: 1 —stabilized body; 2 —outer suspension ring; 3 —gyroscope casing; 4 —gyroscope rotor; 5 —precession-angle sensor; 6 —precession axis; 7 —axis of the gyroscope rotor' s own rotation; 8 —stabilization axis; 9 —driven gear wheel of the reducer; 10 —driving gear wheel of the reducer; 11 —stabilization motor; 12 —amplifier

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Here: A is the equatorial moment of inertia of the gyroscope rotor; H is its own kinetic moment; α and ψ are, respectively, the angles of rotation

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about the stabilization axis of the rotor and the gyroscope casing (together with the outer suspension ring, the stabilized body, and the driven gear of the reducer) with respect to the base; β is the small angle of deflection of the gyroscope casing from the mean position (at which the axis of proper rotation of the rotor is perpendicular to the stabilization axis); K is the stiffness corresponding to the elastic displacement $\alpha - \psi$ of the gyroscope rotor relative to the casing; Ψ is the sum of the moments of inertia of the outer ring, the stabilized body, and the driving gear of the reducer with respect to the stabilization axis; θ is the angle of rotation of the motor rotor reduced to the stabilization axis (i.e., the angle of rotation relative to the stator, reduced by a factor of j, where j is the gear ratio of the reducer); N is the stiffness of the reducer, also referred to the stabilization axis, and Θ is the moment of inertia of the motor rotor together with the driving gear of the reducer, reduced to the stabilization axis (the total moment of inertia of these bodies with respect to the motor-rotor axis, increased j^2 times).

2°. The second differential equation of system (1) can be integrated. As a result we obtain the relation

$$A\,\frac{d\beta}{dt} - H\alpha = 0, \tag{2}$$

in which an arbitrary constant, immaterial for what follows, has been omitted. If now, by means of this relation, the angular velocity $d\beta/dt$ is eliminated from the first equation of (1), we arrive at another form of the system of equations describing the motion of the same idealized conservative system, namely:

$$A\,\frac{d^2\alpha}{dt^2}+\frac{H^2}{A}\alpha=K(\psi-\alpha),\qquad \Psi\,\frac{d^2\psi}{dt^2}=K(\alpha-\psi)+N(\theta-\psi),$$

$$\Theta\,\frac{d^2\theta}{dt^2}=N(\psi-\theta). \tag{3}$$

3°. Let us now consider three masses A, Ψ, and Θ, whose deviations from the equilibrium position we shall denote respectively by α, ψ, and θ (Fig. 2). Let mass A be connected with mass Ψ by a spring of stiffness K, and mass Ψ with mass Θ, in turn, by a spring of stiffness N. Let, moreover, mass A be connected with a fixed origin by a spring whose stiffness is equal to H^2/A. It is obvious that the system of equations describing the oscillations of such a system consists of the same equations as system (3). It follows from this that a frictionless "de-energized" gyroscopic stabilizer admits a mechanical analogy in the form of a system of three successive masses elastically coupled to one another, the first of which is additionally connected by a peculiar "gyroscopic" spring to "inertial" space.

The analogy presented, and similar ones, make it possible to apply to the study of gyroscopic phenomena the techniques and methods of the theory of vibrations of elastic systems. Thus, for example, if in the case of a gyroscopic stabilizer

$$\Psi>\Theta\gg A \tag{4}$$

(as is usually the case in practice), then, as is not difficult to see, the mass A should not have any appreciable influence on the values of the lower natural frequencies of the system under consideration. The lowest frequency ω_1 corresponds to in-phase motion of all three masses (Fig. 3); the next frequency ω_2 corresponds to in-phase motion of the masses A and Ψ, while the motion of mass Θ is in antiphase to them.

The presence of the small mass A in the oscillations of the system with frequencies ω_1 and ω_2 has almost no effect on the magnitude of its displacement α. This displacement can therefore be determined approximately from the first equation of system (3), by omitting in it the term $A\,d^2\alpha/dt^2$. As a result we obtain

$$\alpha=\frac{KA}{KA+H^2}\,\psi. \tag{5}$$

Substitute this expression into the second equation of (3). Then, together with the third equation of (3), we obtain two differential equations with respect to ψ and θ. The corresponding frequency equation has the form

$$\omega^4-\left(\frac{\nu^2\chi^2}{\nu^2+k^2}+n^2+\xi n^2\right)\omega^2+\frac{\nu^2\chi^2 n^2}{\nu^2+k^2}=0. \tag{6}$$

Fig. 2. Mechanical system—an analogue of a de-energized, frictionless gyroscopic stabilizer

Figure 2: Fig. 2. Mechanical system—an analogue of a de-energized, frictionless gyroscopic stabilizer

Fig. 3. Nature of motion of the mechanical system—analogue at the lower oscillation frequencies of the gyroscopic stabilizer

Figure 3: Fig. 3. Nature of motion of the mechanical system—analogue at the lower oscillation frequencies of the gyroscopic stabilizer

The roots of this equation, when conditions (4) are satisfied, differ little from the two smallest roots ω_1 and ω_2 of the frequency equation

$$\omega^6-(\nu^2+k^2+\chi^2+n^2+\xi n^2)\,\omega^4+\left[\nu^2(\chi^2+n^2+\xi n^2)+n^2(k^2+\xi k^2+\chi^2)\right]\omega^2-\nu^2 n^2\chi^2=0 \tag{7}$$

of the system of differential equations (3), or, if the root $\omega = 0$ is left aside, of the original system (1). In equations (6) and (7)

$$\nu = H/A; \qquad k^2 = K/A; \qquad \chi^2 = K/\Psi; \qquad n^2 = N/\Theta; \qquad \xi = \Theta/\Psi. \tag{8}$$

The largest root of equation (7) corresponds to the highest frequency ω_3 of oscillations of the idealized mechanical system. At this frequency

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the masses Ψ and Θ practically remain in place and only the mass A oscillates. Therefore, if in the first equation of the system (3) we put $\psi = 0$, then we ultimately arrive at the formula

$$\omega_3^2 = \frac{1}{A}\left(K + \frac{H^2}{A}\right) = k^2 + \nu^2 \tag{9}$$

for an approximate calculation of the largest root ω_3 of equation (7).

4°. Let us now consider a gyroscopic stabilizer with a connected direct-current motor with independent excitation. To describe the behavior of such a stabilizer, the last of the equations of the system (3) must be replaced by the following three:

Fig. 4. System analogous to a gyroscopic stabilizer with the motor switched on

Figure 4: Fig. 4. System analogous to a gyroscopic stabilizer with the motor switched on

$$\Theta\frac{d^2\theta}{dt^2} = N(\psi - \theta) + \frac{Ci}{g}, \qquad Ri + C\frac{d\theta}{dt} = V, \qquad V = -\mu\Phi(p)\beta. \tag{10}$$

In addition to the notation already encountered above, here: C is the coefficient of the counter-electromotive force of the motor, increased j times; i is the current in the motor armature; R is the ohmic resistance of the motor armature together with the output resistance of the amplifier (the inductive resistance of this circuit is not taken into account here); V is the electromotive force at the amplifier output; μ is the amplification factor of the direct-current voltage; $\Phi(p)$ is the transfer function of the amplifier, normalized so that $\Phi(0) = 1$ (p is the differentiation operator); $g = 9.81$, if the current i is expressed in amperes and the torque on the shaft of the stabilized body is in kilogram-meters.*

An analogue of a gyroscopic stabilizer with a connected motor may be the same mechanical system as for the de-energized sta–

* Note that equations (10), with somewhat modified notation, are also suitable for the case of an alternating-current motor.

stabilizer, but with the addition (Fig. 4) of the "artificial" force

$$P = -\frac{\mu C}{Rg}\Phi(p)\beta \tag{11}$$

and the force of "viscous" resistance

$$F = f\,d\theta/dt \quad (f = C^2/Rg), \tag{12}$$

applied to the mass Θ.

Fig. 4. System analogous to a gyroscopic stabilizer with the motor switched on

To form the "artificial" force P in such a system, an "integrating" element is used, producing, according to law (2), the quantity β, and an amplifier with transfer function $\Phi(p)$. The integrating element and the amplifier here form a feedback loop. Because of the presence of this loop, at large amplification coefficients μ, which are sometimes necessary for technical requirements, the stabilizer may become unstable.

5°. Usually the lower vibration frequencies of a gyroscopic stabilizer change only slightly when the electric circuits of the motor are switched on, and the character of the oscillations of its masses at such frequencies, in comparison with item 3°, remains without essential changes. In a number of cases this circumstance makes it possible to judge the properties of a gyroscopic stabilizer without a preliminary numerical analysis of the complete set of its equations.

Let, for example, the transfer function of the amplifier have the form:

$$\Phi(p) = 1 + p^2T^2, \tag{13}$$

with $\omega_1 T > 1$. Such an amplifier "raises," without changing the phase, the frequencies of the input signal that are higher than ω_1, as a result of which it cannot contribute to increasing the stability of the gyroscopic stabilizer. Indeed, suppose that, with proper switching-on of the motor, the artificial force P, created in accordance with equalities (11) and (13), contributes to damping the oscillations of the stabilizer with frequency ω_1. At this frequency the masses A, Ψ, and Θ, in accordance with what was set forth above, oscillate in phase. However, for oscillations at the next frequency $\omega_2 > \omega_1$, the phases of the oscillations of the masses Θ and A are opposite. Therefore the force P, moreover having a relatively larger amplitude, will now tend to "rock" the system, thereby violating its stability.

6°. Let us note that if, in accordance with item 3°, the term $A\,d^2\alpha/dt^2$ is omitted directly in the first equation of the set (1), then we obtain

$$\alpha = \psi - \frac{H}{K}\frac{d\beta}{dt}. \tag{14}$$

Substituting this expression into the second and third equations (1), we obtain the equations of motion of a conservative gyroscopic system

$$\left(A + \frac{H^2}{K}\right)\frac{d^2\beta}{dt^2} - H\frac{d\psi}{dt} = 0, \quad \Psi\frac{d^2\psi}{dt^2} + H\frac{d\beta}{dt} = N(\theta - \psi), \quad \Theta\frac{d^2\theta}{dt^2} = N(\psi - \theta), \tag{15}$$

whose frequency equation coincides, of course, with equation (6) (not counting the root $\omega = 0$). In the limit, as $K \to \infty$, we arrive at the set of equations for a system with absolutely rigid bearings of the gyro rotor, i.e., at the case $\psi \equiv \alpha$. It can also be obtained directly from the set (1), assuming that $K \to \infty$ and $\alpha \to \infty$, while the moment of the interaction forces of the rotor and housing, $K(\alpha - \psi)$, remains finite.

Equations (15), not differing in form from the equations of the "classical" case, take into account the influence of the elastic compliance of the rotor bearings on the values of the natural frequencies of oscillation of the gyroscopic system.

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CITED LITERATURE

1. A. Yu. Ishlinskii, *Mechanics of Gyroscopic Systems*, Publishing House of the Academy of Sciences of the USSR, 1963.

Note: Figure translations are in progress. See original paper for figures.