THEORY OF ELASTICITY

Corresponding Member of the Academy of Sciences of the USSR A. V. POGORELOV

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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ON THE CRITICAL EXTERNAL PRESSURE ON THE ELLIPSOIDAL BOTTOM OF A CYLINDRICAL RESERVOIR

Cylindrical reservoirs with bottoms in the form of a flattened ellipsoid of revolution are widespread in engineering (Fig. 1). Such reservoirs are often subjected to external pressure, for example when there is a vacuum inside the reservoir. Calculation of the strength of a reservoir under external pressure presupposes investigation of the stability of its bottoms.

When the bottom of a reservoir loses stability and begins to bulge, its capacity to resist external pressure decreases and reaches a certain minimum. This minimum is the lower critical load by definition. If the lower critical load is taken as the design load, loss of stability of the bottom will be excluded. In this connection, determination of the lower critical load is of practical interest. In the present note an appropriate formula is obtained for it.

Fig. 1

We shall proceed from the assumption that the postcritical deformation of the bottom (after loss of stability) possesses axial symmetry. Taking into account the considerations set forth in the author' s preceding works, in particular in (1), we approximate the deformation of the bottom by a mirror-image bulge.

The deformation energy of the bottom is determined by the formula

$$U = 2\pi c E \delta^{5/2} \alpha^{5/2} \rho^{1/2},$$

where δ is the thickness of the bottom; ρ is the radius of the region of bulging; α is the angle between the plane of the curve bounding the region of bulging and the tangent planes of the surface; E is the modulus of elasticity; c is a constant $\simeq 0.19$.

The work performed by the external pressure p in bulging the bottom is

$$A = pV,$$

where V is the change in volume of the reservoir under deformation of the bottom.

We shall characterize the bulging of the bottom by the parameter ρ. Then the condition of elastic equilibrium of the bottom during bulging will be

$$\frac{d}{d\rho}(U - A) = 0.$$

Assuming sufficient flatness of the region of bulging, we shall have

$$d\alpha/d\rho \simeq k,$$

where k is the curvature of the bottom in the original form in a radial section. Hence

$$\frac{dU}{d\rho} = \pi c E \delta^{5/2} \alpha^{5/2} \rho^{-1/2} \left(5\frac{\rho}{\alpha}k + 1\right).$$

Let us denote by h the height of the mirror-reflected segment. Then

$$\frac{dA}{d\rho} = p\frac{dV}{dh}\frac{dh}{d\rho},$$

$$dV/dh = 2\pi\rho^2, \qquad dh/d\rho \simeq \alpha.$$

Thus,

$$dA/d\rho = 2\pi p \rho^2 \alpha.$$

Substituting the obtained values of $dU/d\rho$ and $dA/d\rho$ into the equilibrium equation, we obtain the relation between the perceived pressure p and the deformation of the bottom:

$$p = \frac{1}{2} c E \left(\frac{\delta}{\rho}\right)^{5/2} \alpha^{3/2} \left(5\frac{\rho}{a}k + 1\right).$$

Let us introduce, instead of ρ, the parameter $\xi = \rho/R$, where R is the radius of the base of the bottom. As a function of this parameter, the quantities α and k are expressed by the formulas

$$\alpha \simeq \operatorname{tg}\alpha = \frac{\lambda\xi}{(1-\xi^2)^{1/2}}, \qquad k \simeq \frac{\lambda}{R}\frac{1}{(1-\xi^2)^{3/2}},$$

where λ is the ratio of the height of the bottom to the radius of the base, i.e., the ratio of the minor semi-axis of the ellipsoid to the major semi-axis. Substituting the found values of α and k, as functions of ξ, into the formula for p, we obtain

$$p = \frac{c}{2}E\left(\frac{\delta}{R}\right)^{5/2}\lambda^{3/2}\theta(\xi),$$

$$\theta(\xi) = \frac{1}{\xi}\frac{1}{(1-\xi^2)^{3/4}}\left(\frac{5}{1-\xi^2}+1\right).$$

The lower critical pressure p_i corresponds to the smallest value of θ. It is obtained at $\xi \simeq 0.5$ and is equal to $\simeq 18.8$. Hence, taking into account the value of the constant $c \simeq 0.19$, we obtain the formula for the lower critical pressure

$$p_i = 1.8E(\delta/R)^{5/2}\lambda^{3/2}.$$

In the course of our derivation we assumed in advance that α is small. Let us show that this assumption is satisfied if λ is sufficiently small. Indeed,

$$\alpha \simeq \lambda\xi/(1-\xi^2)^{1/2}.$$

For $\xi = 0.5$

$$\alpha \simeq 0.5\lambda$$

and, consequently, is small together with λ. It may be assumed that the condition of smallness of α is fulfilled if $\lambda < 0.5$.

Physical-Technical Institute
of Low Temperatures
Academy of Sciences of the Ukrainian SSR

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REFERENCES

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Note: Figure translations are in progress. See original paper for figures.