ON A METHOD FOR DETERMINING THE DYNAMIC STRESS-STRAIN RELATION WITH AN INFLECTION POINT

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Fig. 1

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Abstract

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ON A METHOD FOR DETERMINING THE DYNAMIC STRESS-STRAIN RELATION WITH AN INFLECTION POINT

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In ([1]), on the basis of the theory of transverse impact, the possibility was shown of developing a construction of dynamic diagrams $\sigma(\varepsilon)$ for various materials. In the present work a procedure is given for determining the dynamic stress-strain relation $\sigma(\varepsilon)$, taking into account the presence of an inflection point characteristic of certain polymeric materials, and results are presented of experimental investigations on a pneumatic impact tester for the industrial rubber "Shifr 65."

1. Let a body having velocity v_0 instantaneously come into contact with a rectilinear flexible thread at a right angle. Let us consider the picture of the motion of the thread. Figure 1 shows changes in the thickness of the cross section of the thread and the corresponding wave diagram in the (x, t) plane, as well as the form of the stress-strain curve having an inflection point. In regions I, II, III the parameters of the thread will be denoted by the indices $1, 2, 3$, respectively. Region 0 corresponds to the undisturbed part of the thread. Let u_i, ε_i, T_i denote respectively the particle velocity, the strain, and the tension in the i-th region; let b and c denote the propagation velocities of the transverse and longitudinal waves of strong discontinuity. In region I, where Riemann waves occur,

$$u_1 = \int_0^{\varepsilon_1} \sqrt{\frac{1}{\rho_0} \frac{dT}{d\varepsilon}}\, d\varepsilon. \tag{1}$$

Fig. 1

Writing, for each of the indicated regions, the dynamic and kinematic compatibility conditions, we obtain, at the longitudinal strong discontinuity, i.e., at the boundary of regions I and II, the law of conservation of mass and the equation of momentum:

Fig. 2

Figure 2: Fig. 2

$$(c+u_1)(1+\varepsilon_2)=(c+u_2)(1+\varepsilon_1), \tag{2}$$

$$\rho_0(c+u_1)(u_2-u_1)=(T_2-T_1)(1+\varepsilon_1). \tag{3}$$

In the neighborhood of the kink point, according to ([1]), we have:

$$\rho_0(b+u_2)^2=(1+\varepsilon_2)T_2; \tag{4}$$

$$b=v_0\operatorname{ctg}\gamma; \tag{5}$$

$$u_2=b(1-\cos\gamma)/\cos\gamma. \tag{6}$$

In addition, we regard the dependence $T_1=T_1(\varepsilon_1)$ up to the point of inflection as known (see (2)); $v_0(\gamma)$ is determined from experiment; $T_2=T_2(\varepsilon_2)$ is the desired relation beyond the point of inflection. Let us show that, if $v_0=v_0(\gamma)$ is known from experiment, then the dependence $T_2=T_2(\varepsilon)$ under dynamic loading can be determined. Indeed, expressing all unknown quantities through the parameter γ from (4), (5), and (6), we have

$$T_2(1+\varepsilon_2)=\varphi(\gamma). \tag{7}$$

Fig. 2. Dependence of v_0 on γ. 1 –for polyvinyl chloride, $d=1$ mm; 2 –for rubber, $d=5$ mm

Taking into account that

$$(T_2-T_1)/(\varepsilon_2-\varepsilon_1)=(dT/d\varepsilon)_{\varepsilon=\varepsilon_1}=\psi(\varepsilon_1)$$

(3), we shall have

$$[T_1+\psi(\varepsilon_1)(\varepsilon_2-\varepsilon_1)](1+\varepsilon_2)=\varphi(\gamma). \tag{8}$$

On the other hand, from (1), (2), and (3) we obtain

$$u_2(\gamma)-u_1(\varepsilon_1)=\sqrt{\psi(\varepsilon_1)/\rho_0}\,(\varepsilon_2-\varepsilon_1). \tag{9}$$

Thus the posed problem is mathematically reduced to solving the system of nonlinear equations (8), (9) with respect to ε_1 and ε_2. Consequently, from (7) T_2 will be determined as a function of γ. Thus, we obtain parametric equations for the curve $T_2(\varepsilon_2)$.

2. To illustrate the indicated method, tests were carried out on a rubber cord "Shifr 65" with diameter 5 mm on a pneumatic apparatus. The motion of the cord was recorded with an SKS-1 high-speed motion-picture camera. Experiments on the rubber cord were conducted in the velocity range from 2 to 170 m/sec. From the motion-picture frames of the rubber cord motion, the values of the kink angle γ were measured for each impact velocity. On the basis of these data, an experimental dependence of the impact velocity v_0 on the kink angle γ was obtained; its graph is shown in Fig. 2.

Approximating the dependence $v_0 = v_0(\gamma)$ with acceptable accuracy by the function

$$v_0 = (15.8\gamma^2 + 12.526\gamma)/(1.570796 - \gamma),$$

we performed calculations up to the point of inflection (2) and beyond the point of inflection by formulas (7), (8), and (9). As a result of processing the data by the method set forth, the complete dynamic diagram $T(\varepsilon)$ was obtained for the rubber "Shifr 65" up to the dynamic rupture limit (see Fig. 3).

To find the point of inflection, we examine the behavior of the first derivative $dT/d\varepsilon$ as a function of ε. From the graph of $dT/d\varepsilon$, the point of inflection can be found with sufficient accuracy. The point of inflection for the material under investigation corresponds to the value $\gamma = 46^\circ 30'$; in this case the deformation and tension are, respectively, $\varepsilon = 0.1215$, $T = 2.548$ kgf.

To determine the deformation ε_1 before the point of inflection, it is necessary to find an additional dependence between γ and γ_1. This dependence is determined as follows: assigning arbitrary values of γ_1 beyond the inflection point, by selecting the corresponding γ up to the inflection point we achieve satisfaction of the equality

$$\sqrt{\rho_0 \psi(\varepsilon_1)}(u_2 - u_1) = \frac{\varphi(\gamma_1)}{1 + \varepsilon_1 + (u_2 - u_1)/\sqrt{\psi(\varepsilon_1)/\rho_0}} - T_1(\varepsilon_1),$$

obtained from (8) and (9). Thus, knowing the value γ_1, we determine the corresponding ε_2 beyond the inflection point and the corresponding T_2 from formula (7).

Figure 3 also gives the static loading diagram for the same rubber. The dynamic loading curve has a form similar to the static one and lies above the static curve; in the initial portion, for deformations up to 0.03, the dynamic curve passes above the static one by almost a factor of 2, and thereafter by a factor of $1.5 \div 2$.

Fig. 3. Dynamic (1) and graphical (2) diagrams $T(\varepsilon)$

Fig. 4. Dependence of T on t

The tension corresponding to dynamic rupture is twice the tension at static rupture. For the rubber studied, the dynamic strength limit is 28.42 kgf, while the static limit is 13–14 kgf.

3. To check the results obtained, direct measurements were made of the deformation in the impact zone and of the tension at the point where the thread was fastened. Direct measurement of the deformations was carried out by applying marks to the rubber in the impact zone, visible in the motion-picture frames, i.e., the deformations were found from the displacements of the marks. The tension at the place where the cord was fastened was measured as follows: one end of the cord is attached to a metal beam having a natural frequency of oscillation of at least 2000 Hz. A wire transducer is glued to the beam. The bending of the beam is recorded by a strain-gauge installation. By processing the oscillograms for each impact velocity, we obtain the experimental dependence $T(t)$ at the point of rigid fastening.

The measurements carried out made it possible to determine the range of applicability of the laws of elastoplastic deformation for the material studied. For this purpose, the problem of reflection of plastic waves from the fixed end of a thread of finite length under transverse impact was solved. The problem was solved by the method of characteristics for impact velocities up to $v_0 = 27$ m/sec (corresponding to the inflection point), and the dependence $T = T(t)$ at the fastening point was obtained.

The results of the calculations are shown in Fig. 4, where the solid curve corresponds to the calculated data and the dashed curves to the experimental data. As can be seen from the figure, the experimental curves $T(t)$ coincide with the calculat-

···at low impact velocities (up to $v_0 = 15$ m/sec). This indicates the applicability of the elastic-plastic law of deformation to the rubber under investigation. For higher velocities ($v_0 > 15$ m/sec), the tension T cannot be regarded as a function of ε alone.

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Note: Figure translations are in progress. See original paper for figures.