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Abstract

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ON TURBULENT-GRAVITATIONAL COAGULATION OF CLOUD DROPLETS

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In numerous studies of the mechanism of formation of cloud droplets, difficulties have been encountered in explaining the appearance, in a relatively short time (of the order of 20-30 min), of a sufficiently broad spectrum of droplet sizes (with diameters from 1 to 30-40 μ). Condensational growth of droplets leads to a narrow spectrum, while coagulation growth, according to existing theoretical ideas, is either forbidden up to $d = 30\text{-}40\,\mu$ (purely gravitational coagulation ([1–3])), or, for the mean charge values existing in clouds, is effective only up to $d \approx 5\text{-}7\,\mu$ (electrostatic gravitational coagulation ([4])). The mechanism by which turbulence affects the coagulation of cloud droplets has been studied by a number of authors, who estimated the rate of coagulation growth to within unknown constant coefficients ([5–8]). Recently, Yu. S. Sedunov carried out exact calculations of the mutual velocity of aerosol particles in a turbulent flow ([9]), and on the basis of these calculations the coefficients of turbulent diffusion were determined in finite form ([9a,b]). Determination, with the aid of work ([9]), of the growth rate of cloud droplets due to turbulent diffusion and the known turbulent "acceleration mechanism" once again confirmed that they alone (for experimentally determined values in clouds of ε—the rate of dissipation of turbulent energy) cannot determine the rates of formation of the cloud-droplet spectrum observed in nature ([9b]). This circumstance is connected with the fact that at small distances between droplets turbulent coagulation is small (it is effective at large distances, since the coefficient of mutual turbulent diffusion $D = ar^2$, where r is the distance between the centers of the particles). At the same time, for other mechanisms of coagulation the opposite phenomenon is characteristic—they are ineffective at large distances.

Therefore we have attempted to estimate the joint influence of turbulent and electrostatic coagulation on the growth rate of cloud droplets. The estimate was made for a two-layer model of the phenomenon, in which the space outside a large droplet is divided into two parts by a sphere located from the droplet at a distance equal to the turbulent mean free path of the particles*. Outside the boundary sphere the principal mechanism of approach is the turbulent one. It ensures a rapid approach to this sphere of small particles with comparatively large relative velocities, and therefore here the effect of the remaining mechanisms of approach may be neglected. Inside the boundary sphere, where the kinetic approximation may be used, motion occurs owing to the forces of gravity, electrostatic attraction, and hydrodynamic interaction, while the contribution of turbulent mutual motion is negligibly small. Since inside the boundary sphere the motion occurs at distances between droplet surfaces Δ comparable with the droplet sizes,

* By the mean free path we understand the characteristic mean distance over which particles in a turbulent flow must separate so that the relative velocities of the particles at the initial and final instants are uncorrelated. The quantity determining this distance in the collision process is $l_T \approx 1.5(R_1 + R_2)$, where R_1 and R_2 are the radii of the larger and smaller particles ([9b]).

then, for a correct formulation of the problem, a careful consideration of the expressions for the aforementioned forces is necessary.

For the electrostatic interaction of two charged drops we used the expression described in detail in ([4]). The series for the capacitance coefficients (([4]), p. 55, p. 241) and the corresponding derivatives determining the interaction forces converge sufficiently rapidly even at distances $\Delta \sim 0.001 R_2$. This made it possible to program efficiently the expression for the electrostatic forces on the BESM-2 computer.

The forces of hydrodynamic interaction were considered in the Stokes approximation ([10a]), since, according to the formulation of the problem, we are dealing with particles with $R_1 < 20 \div 30\,\mu$ and with $\Delta < 2 \div 5 R_1$. As is known, in the Stokes approximation the forces $\mathbf{f}_1$ and $\mathbf{f}_2$ acting on the particles are linearly related to the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ of the particles relative to the air. If one considers plane motion of drops, for example, in a vertical plane, and introduces a Cartesian coordinate system connected with the line of centers (the Oz axis along the line of centers, the Ox axis perpendicular to it), these relations can be represented in the form:

$$f_{ix} = (a_x^{i1} v_{1x} + a_x^{i2} v_{2x})\, 6\pi\eta R_i, \qquad f_{iz} = (a_z^{i1} v_{1z} + a_z^{i2} v_{2z})\, 6\pi\eta R_i, \qquad i = 1, 2, \tag{a}$$

where a_x^{ik} and a_z^{ik} are functions of the distance between the centers of the drops r and of the radii R_i.

In the case when the motion takes place along the line of centers ($v_{ix} = 0$), the problem is axisymmetric and an exact solution exists for a^{ik} ([10b,12]). In the general case, however, a number of authors have found expressions for these functions in the form of series in powers of R_i/r ([3,11]). These series, generally speaking, converge poorly for small distances Δ between drops. We have noted that if one uses the relation inverse to (a), having the form

$$v_{ix} = b_x^{i1} \frac{f_{1x}}{6\pi\eta R_1} + b_x^{i2} \frac{f_{2x}}{6\pi\eta R_2}, \qquad v_{iz} = b_z^{i1} \frac{f_{1z}}{6\pi\eta R_1} + b_z^{i2} \frac{f_{2z}}{6\pi\eta R_2}, \qquad i = 1, 2, \tag{b}$$

then the analogous series for b^{ik} converge better than the series for a^{ik}. This allowed us, within the same approximation as that of Hocking ($O(r^{-7})$ ([3])), to obtain for b^{ik} expansions in series in powers of R_0/r in the form

$$-b_x^{11} = 1 - \frac{3}{4}\frac{R_1 R_2^3}{r^4} - \frac{17}{16}\frac{R_1 R_2^5}{r^6};$$

$$-b_x^{12} = \frac{3}{4}\frac{R_2}{r} + \frac{R_2(R_1^2 + R_2^2)}{4r^3} + \frac{3}{8}\frac{R_1^3 R_2^4}{r^7}; \qquad \text{(c)}$$

$$-b_z^{11} = 1 - \frac{15}{4}\frac{R_1 R_2^3}{r^4} - 2\frac{R_1 R_2^5}{r^6} + \frac{15}{2}\frac{R_1^3 R_2^3}{r^6};$$

$$-b_z^{12} = \frac{3}{2}\frac{R_2}{r} - \frac{R_2(R_1^2 + R_2^2)}{2r^3} + \frac{75}{4}\frac{R_1^3 R_2^4}{r^7};$$

b^{21} and b^{22} are obtained respectively from b^{12} and b^{11} by transposing R_1 and R_2.

Taking the expression inverse to (b), with account of (c), we obtain expressions for a^{ik} that converge for small Δ better than Hocking' s, and the convergence proved to be substantially improved for drops of comparable sizes: $R_2 > 0.5R_1$. The improvement in convergence is connected with the fact that, when seeking the expression inverse to (b), in order to obtain (a) we computed the determinants $|b^{ik}|$ exactly, rather than limiting ourselves to terms $O(r^{-7})$, as Hocking did.

In Table 1 the above is illustrated for the case of equal drops moving along the line of centers. It gives the values $\lambda = |a_z^{11} + a_z^{12}|$, characterizing the ratio of the force acting on one of the equal drops moving with the same velocity v_z, at a distance Δ between the surfaces, to the force acting on an isolated drop at the same velocity of motion. The last row contains the values calculated by us.

In accordance with what has been said, we considered the system of dimensionless equations:

$$k\frac{dw_1}{dt} = 1 - \frac{zL_1 - xT_1}{r} + a^3\alpha\frac{zf(r)}{r^3}; \qquad (1)$$

$$k\frac{du_1}{dt} = -\frac{xL_1 + zT_1}{r} + a^3\alpha\frac{xf(r)}{r^3}; \qquad (2)$$

$$a^2k\frac{dw_2}{dt} = a^2 - \frac{zL_2 - xT_2}{r} + a^2\alpha\frac{zf(r)}{r^3}; \qquad (3)$$

Fig. 1

Figure 1: Fig. 1

$$a^2k\frac{du_2}{dt} = -\frac{xL_2 + zT_2}{r} + a^2\alpha\frac{xf(r)}{r^3}; \tag{4}$$

$$\frac{dx}{dt} = u_2 - u_1; \tag{5}$$

$$\frac{dz}{dt} = w_2 - w_1, \tag{6}$$

where $z = z_2 - z_1$; $x = x_2 - x_1$; $r^2 = x^2 + y^2$; $u_i = dx_i/dt$; $w_i = dz_i/dt$; x_1, z_1, x_2, z_2 are the Cartesian coordinates of the centers of the drops (the axis Oz is vertical), $f(r)$ is defined in [4] (§ 11 and Appendix IV), $k = 4\rho^2 R_1^3 g/81\eta^2$; $\alpha = q_1 q_2/6\pi\eta R_1^3 a^3 v_{1s}$; $a = R_2/R_1 \leq 1$; q_1 and q_2 are the charges of the drops; ρ is their density; η is the coefficient of viscosity of air; g is the acceleration of gravity; v_{1s} is the sedimentation velocity of the large drop. The functions L_i, T_i, linear with respect to u_k, w_k, have essentially the same form as in Hocking [3]. The difference, in accordance with what has been said, consists only in the fact that the denominators of these expressions contain the corresponding determinants composed of the coefficients b^{ik} of equation (b) (in Hocking these determinants are given with accuracy to $O(r^{-7})$). The system of equations (1)–(6) was solved on a BESM-2 computer with the initial conditions:

$$\text{at } t = 0 \qquad x = x_0;\ z = z_0;\ w_{10} = 1;$$

$$u_{10} = 0;\ w_{20} = a^2;\ u_{20} = 0.$$

Fig. 1

Finding, for the limiting trajectory, the initial horizontal separation of the drops $x_{0\text{pr}}$, at which the trajectory of the small drop touched a sphere of radius $R_1 + R_2$ (or $1 + a$ in dimensionless variables), we determined the capture coefficient $E = x_{0\text{pr}}^2$. Taking the initial vertical separation of the drops to be very large ($z_0 = 50$), we determined the ordinary capture coefficient of gravitational coagulation E_{g}. Taking instead $z_0 = l_{\text{T}} + R_1 + R_2 = 2.5(R_1 + R_2) = 2.5(1 + a)$, we found the capture coefficient E_{Tg} for our two-layer model of turbulent-gravitational coagulation. Figure 1 presents the computed values of E_{Tg} (curves 1, 4, 5, points 6, 7) and E_{g} (curves 2, 3). Curves 1, 2, 3 correspond to $k = 23.3$ ($a_1 = 25\,\mu$, $\eta = 1.8 \cdot 10^{-4}$ poise). Curve 2 was computed according to Hocking, curve

3 according to our refined formulas. Curves 4, 5 and points 6, 7 correspond to $k = 11.9$ ($a_1 = 20\,\mu$, $\eta = 1.8 \cdot 10^{-4}$ poise). For this value $k = 11.9$, over the same range of values of a ($0.45 \div 0.75$), the coefficient $E_g < 10^{-4}$. Curve 4 corresponds to $\alpha = 0$, curve 5 to $\alpha = \bar{\alpha}$, calculated for the case of oppositely charged drops having the mean values of charges observed in clouds, $|q| = 10^{-4} R$ [4]. Points 6 and 7 correspond to $\alpha = 10\bar{\alpha}$ and $\alpha = 100\bar{\alpha}$. Figure 1 shows that turbulent-gravitational coagulation does not substantially change the magnitudes of the critical values of a outside which coagulation is impossible ($E = 0$).

Average electric charges practically do not affect the magnitude of E^*.

In view of the fact that, in the model considered, under the adopted boundary conditions the turbulence of the flow ensured that drops approached one another at close range with large relative velocities, we may conclude that turbulent diffusion, even together with gravitational and electrostatic coagulation, cannot ensure the coagulation growth of cloud drops with $R < 18 \div 19\,\mu$. The result obtained once again justifies the model of the phenomenon adopted by us.

Table 1

Δ/R	0	0.1	0.2	0.5	0.8	1.0
λ, exact	0.645	0.655	0.660	0.675	0.692	0.705
λ according to Hocking	0.257	0.440	0.525	0.636	0.675	0.695
λ	0.617	0.630	0.642	0.670	0.688	0.705

It seems to us that the formation of a sufficiently broad spectrum of cloud drops is associated with condensation processes occurring under conditions of fluctuations of the principal cloud parameters: flow velocity, supersaturation, temperature, etc. This circumstance requires special investigations.

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CITED LITERATURE

1. I. Langmuir, *J. Meteorol.*, **5**, 175 (1948).

2. H. S. Shishkin, *Clouds, Precipitation, Atmospheric Electricity*, 1954.

3. L. M. Hocking, *Quart. J. Roy. Met. Soc.*, **85**, No. 3 (1959).

4. L. M. Levin, *Investigations in the Physics of Coarsely Dispersed Aerosols*, Publishing House of the Academy of Sciences of the USSR, 1961.

5. V. G. Levich, *DAN*, a) **99**, No. 5 (1954); b) **99**, No. 6 (1954).

6. T. W. East, T. S. Marshall, *Quart. J. Roy. Met. Soc.*, **80**, No. 343 (1954).

7. P. G. Saffman, T. S. Turner, *J. Fluid Mech.*, **1**, No. 1 (1956).

8. A. I. Ivanovskii, I. P. Mazin, *Tr. TsAO*, issue 35 (1961).

9. Yu. S. Sedunov, *Izv. AN SSSR, ser. geofiz.*, a) No. 5 (1963); b) No. 11 (1963); c) No. 1 (1964).

10. S. V. Shenai-Severin, *Izv. AN SSSR, ser. geofiz.*, a) No. 8 (1957); b) No. 10 (1958).

11. G. T. Kynch, *J. Fluid Mech.*, **5**, 193 (1959).

12. M. Stimson, G. B. Jeffery, *Proc. Roy. Soc.*, A, **111**, 110 (1926).

* The case $a \approx 1$ requires additional consideration; however, it cannot change the conclusions given above.

Note: Figure translations are in progress. See original paper for figures.