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Abstract

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LIMITING CROSS SECTION OF THE RADIATION BEAM OF AN OPTICAL QUANTUM GENERATOR

1. The article considers limitations on the cross section of the radiation beam of optical quantum generators. In continuous-operation optical generators, for very large transverse dimensions of the resonator, limitation of the radiation-beam cross section is possible in principle because of delay in the interaction of remote regions of the generator or because of detuning of the natural frequencies of individual parts of the resonator. In pulsed optical generators (generators with Q-switching ([1,2])) the limitation of the cross section arises because the process of generation of radiation by the medium occurs over a time of the order of the time required for establishing the modes of oscillation of the resonator.

2. Continuous regime. Suppose that in a quantum generator the generation region has accidentally split into two mutually incoherent parts, each of which is in itself a separate quantum generator or, for brevity, a "subgenerator." If the initial generation regime was stable, then, owing to diffractive exchange of the electromagnetic field, locking of the subgenerators occurs. The delay in the field exchange may be taken into account by introducing an effective delay time τ of the field entering one subgenerator from the other. Then the problem reduces to considering generation regimes in a system of two coupled generators ([5]), with the field equations ([5]) replaced by the following:

$$\ddot{E}_i(t) + \frac{\omega_i}{Q_0}\dot{E}_i(t) + \omega_0\alpha\dot{E}_j(t-\tau) + \omega_i^2 E_i(t) = -4\pi\ddot{P}_i(t), \tag{1}$$

where the indices i, j refer to the first and second subgenerators; E is the electric-field strength; Q_0 is the Q-factor of the oscillation modes of the subgenerators*; ω_0 is the frequency of the working transition of the active medium; ω_i are the natural frequencies of the oscillation modes of the subgenerators, which are assumed to be distributed symmetrically with respect to the frequency ω_0 by an amount $\omega_0\Delta$, not exceeding the linewidth of the resonator; P is the polarization of the active medium.

Investigation of stability with allowance for delay, in addition to the conditions found in ([5]), ($\alpha < 0,\ |\alpha| > \Delta$), gives an additional condition on the magnitude of the delay:

$$|\alpha|\tau \lesssim 1. \tag{2}$$

The coefficient of diffractive coupling α is determined ([5]) as the difference between the damping rates of the initial oscillation mode and the oscillation modes arising in the subgenerators. If, for definiteness, one assumes that the initial oscillation mode is the lowest oscillation mode TEM_{00} of a resonator with plane square mirrors of size $2a$, and that in the subgenerators the lowest oscillation modes of a resonator with rectangular plane mirrors of size $a \times 2a$ are excited, then, for calculating the diffraction losses, one may use the formulas of L. A. Vainshtein ([3]). It should be noted that, although such a choice is not entirely unique, it describes all the essential features of the diffractive coupling of two subgenerators.

* For simplicity it has been assumed that identical oscillation modes are excited in the subgenerators.

In this approximation, according to (4):

$$\alpha = \frac{\lambda}{L}\pi\beta \left\{ \frac{\sqrt{2\pi N} + \beta}{\left[(\sqrt{2\pi N} + \beta)^2 + \beta^2\right]^2} - \frac{\sqrt{\frac{1}{2}\pi N} + \beta}{\left[(\sqrt{\frac{1}{2}\pi N} + \beta)^2 + \beta^2\right]^2} \right\}, \tag{3}$$

where $N = a^2/L\lambda,\ \beta = 0.824$.

The condition $\alpha < 0$ is always satisfied, and since $\alpha \sim a^{-3} \sim \tau^{-3}$, condition (2) is also satisfied for arbitrarily large values of the delay τ. Consequently, in continuous optical quantum generators the delay effect does not limit the generation region. However, condition (2) may impose substantial limitations in cases where the coupling of the generators is not diffractional and does not decrease so strongly with increasing τ. In diffraction-coupled continuous generators the limiting condition is $|\alpha| > \Delta$. If the detuning of the natural frequencies of the subgenerators is caused by a difference δL in the optical paths between the mirrors in the subgenerators, then this condition reduces to $(\delta L/\lambda) < 2(L/\lambda)|\alpha|$, i.e., $\delta L/\lambda$ determines the Fresnel number $N_{\max}$ of the limiting generation region. For example, at $\lambda = 1\,\mu$ and $\delta L \simeq 1\,\text{Å}$, which is close to the limit of quality for plane mirrors, from (3) we find $N_{\max} \simeq 800$, or, for $L = 100$ cm, the limiting transverse size of the generation region is $2a \simeq 5$ cm.

3. **Pulsed regime.** The development of the generation region in pulsed quantum generators may proceed in two ways: a) the appearance and growth of separate mutually incoherent "streams," b) the merging of neighboring streams with the establishment of coherence of their radiation owing to diffractional exchange of the field. Since the rate of these processes

is finite, it is natural to expect a limitation of the generation region in the pulsed regime. Let us consider these two processes separately.

a) The minimum size of the initial generation region is determined by the magnitude of the diffraction losses $\Gamma_{\max}$ compatible with the condition of self-excitation:

$$\Gamma_{\max} = 1 - [\eta(1-r)]^{-1},$$

where r and η are the values of the nondiffraction losses and the single-pass gain, respectively. For example, in ruby quantum generators with modulated Q, $(1-r)\eta \simeq 10$. Therefore $\Gamma_{\max} \simeq 0.9$, which corresponds to the value of the Fresnel number of the generation region $N_0 = a_0^2/L\lambda \sim 0.1$, where a_0 is the radius of the initial generation region.

If $E_0(x)$ is the field distribution* on the mirrors $(-b < x < b)$ at the moment generation begins $(t = 0)$, then the change of the field at subsequent times is given by the expression

$$E(x,t) = \exp[-ikct] \sum_m A_m U_m(x) \exp\left[-i\frac{k_x^2}{2k}ct\right], \tag{4}$$

where $k = 2\pi/\lambda$, λ is the wavelength of the generated radiation. Expression (4) represents an expansion of the field in the eigenfunctions of the resonator, which describe the field distribution along the plane mirrors of an open resonator. According to L. A. Vainshtein (4):

$$U_m(x) = \frac{1}{\sqrt{2b}} \exp[ik_x x], \qquad k_x = \frac{m\pi}{b(1+\beta(1+i)/M_b)}, \tag{5}$$

where $M_b = \sqrt{2kb^2/L}$, $\beta = 0.824$. It follows from (4)–(5) that, formally, diffusion of the field $E(x,t)$ is completely analogous to the spreading of the wave packet of a particle located in a potential well of size $2b$. For $M_b \gg 1$, which corresponds to real generators, the mass of such a particle is $m \simeq \hbar k/c$. Therefore the radius of the generation region changes according to a law close to the following–

* For simplicity, the one-dimensional case is considered, since the two-dimensional case leads to analogous results.

following:

$$a(t) = a_0 \left[1 + \left(\frac{c\lambda t}{2\sqrt{\pi a_0^3}}\right)^2\right]^{1/2}. \tag{6}$$

The range of applicability of (6) is $a \ll b$.

For example, in a pulsed oscillator with $L = 50$ cm and $(1 - r)\eta = 10$ at $\lambda = 7 \cdot 10^{-5}$ cm ($a_0 = (0.1\, L\lambda)^{1/2} \simeq 0.2$ mm), over a time $t_0 \simeq 10^{-8}$ sec, which is characteristic for the development of a pulse in oscillators with modulated Q, the generation region spreads to a size of the order of 6 mm.

b) We shall also consider the merging of generation regions for the case of an oscillator with modulated Q. Analyzing the oscillation equations (3), we obtain the following equations for the establishment of the phases φ_i of the fields in the suboscillators:

$$\dot{\varphi}_1 - \dot{\varphi}_2 = \alpha\omega_0 \sin(\varphi_1 - \varphi_2). \tag{7}$$

It follows from (7) that the establishment of the phases of the suboscillators is described by the expression $\operatorname{tg} \frac{\varphi_1 - \varphi_2}{2} = C \exp(-|\alpha|\omega_0 t)$. Phase locking, meaning the joint generation of coherent radiation, occurs over a time

$$t_0 = (|\alpha|\omega_0)^{-1}. \tag{8}$$

Since the coupling coefficient α depends substantially on the generation region, this condition imposes a restriction on the limiting cross-sectional size of the coherent beam of a pulsed quantum oscillator that can be obtained by merging neighboring generation regions. For a ruby oscillator with the preceding parameters, over a time $t_0 \simeq 10^{-8}$ sec, taking into account that the magnitude of the diffraction coupling coefficient is close to that given by expression (3), we find a generation region of the order of 4 mm.

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Note: Figure translations are in progress. See original paper for figures.