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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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On the Question of Temperature Stresses in Nonuniformly Heated Gases

As is known, for a gas in a state of equilibrium Pascal' s law is valid, according to which the pressure at a given point is the same in all directions. However, the thermodynamic derivation of this law loses its force in the presence of a temperature gradient.

To determine what deviations from Pascal' s law should then be observed, one may proceed on the basis of the kinetic theory of gases. As was shown earlier ($^{1-4}$), temperature stresses in nonuniformly heated gases must be the principal cause of such important phenomena as thermophoresis of particles and thermal transpiration of gases in capillaries. The first molecular-kinetic analysis of the problem was given by Maxwell, who showed (^{5}) that nonuniformity of temperature in a gas leads to a change in the stress tensor. However, in deriving the tensor of temperature stresses, Maxwell assumed that terms proportional to the square of the temperature gradient are small in comparison with terms containing second derivatives of the temperature. This assumption is incorrect even for an arbitrarily small temperature gradient in the simplest case of a one-dimensional temperature field with everywhere the same intensity (density) of the heat flux. The latter assertion can be derived by considering, for this case, the stress tensor in a gas in the third approximation of the Chapman—Enskog theory (^{6}). As will be shown in the present article, the corresponding rigorous expression for the stress tensor, retaining terms quadratic with respect to the temperature gradient, leads to isotropy of the tensor of temperature stresses. Maxwell' s theory in this case leads to anisotropy of the pressure in different directions. This difference makes it possible to decide experimentally which of the two theories is correct.

Fig. 1

Let us consider a narrow infinite slit of width d, filled with gas (see Fig. 1). The slit is formed by two infinite planes. Suppose that the upper plane has temperature T_1, and the lower one T_2; moreover $T_1 > T_2$, which excludes the possibility of gravitational convection. It is evident that the gas will be at rest, and, by virtue of the law of conservation of momentum, there will be no pressure gradient in any direction. Choose the x-axis in the direction perpendicular to the plane of the plates, and the axes z and y parallel to the plates. In the direction of the x-axis there is a temperature gradient equal to $\partial T/\partial x$, and, by virtue of the law of conservation of energy, a constant heat flux Q, equal to

$$Q = -\varkappa\, \partial T/\partial x, \tag{1}$$

where $\varkappa$ is the thermal-conductivity coefficient of the gas.

The tensor of viscous and thermal stresses in the third approximation of the Chapman—Enskog theory can be represented in the form:

$$P = P^{(0)} + P^{(1)} + P^{(2)}, \tag{2}$$

where, for our case,

$$P^{(0)} = p \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}, \tag{3}$$

$$P^{(1)} = 0; \tag{4}$$

$$P^{(2)} = \frac{1}{3}\bar{\omega}_3 \frac{\eta^2 R}{p\mu} \frac{\partial^2 T}{\partial x^2} \begin{vmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix} + \frac{1}{3}\bar{\omega}_5 \frac{\eta^2 R}{p\mu T} \left(\frac{\partial T}{\partial x}\right)^2 \begin{vmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{vmatrix}; \tag{5}$$

$P^{(1)}$ is the second approximation of the theory, $P^{(2)}$ the third. Expressions (4) and (5) are obtained from the general tensor of viscous and thermal stresses after very cumbersome calculations with a limiting transition to the case when

$$\partial p/\partial x = \partial p/\partial y = \partial p/\partial z = 0; \tag{6}$$

$$u = v = w = 0; \tag{7}$$

$$\partial T/\partial y = \partial T/\partial z = 0. \tag{8}$$

In formulas (3)—(8) the following notation has been introduced: η is the viscosity; p is the pressure; μ is the molecular weight; R is the universal gas constant;

u, v, and w are the components of the gas velocity in the directions of the axes x, y, and z. The coefficients $\bar{\omega}_3$ and $\bar{\omega}_5$, according to the results of Chapman (6), have the form

$$\bar{\omega}_3 = 3; \tag{9}$$

$$\bar{\omega}_5 = 3(T/\eta)\partial\eta/\partial T. \tag{10}$$

Result (5) represents the third approximation in full form, i.e., taking into account the squares and products of the first derivatives of the temperature. Let us give the normal components of the pressure tensor in the directions of the axes x and z, according to Maxwell' s results, in the state when $u = v = w = 0$:

$$P_{xx} = p + \frac{3\eta^2 R}{p\mu}\frac{\partial^2 T}{\partial x^2} + \frac{3}{2}\frac{\eta^2 R}{p\mu}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right); \tag{11}$$

$$P_{zz} = p + \frac{3\eta^2 R}{p\mu}\frac{\partial^2 T}{\partial z^2} + \frac{3}{2}\frac{\eta^2 R}{p\mu}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right). \tag{12}$$

For the case of the slit considered, relations (11) and (12) take the form

$$P_{xx} = p + \frac{9}{2}\frac{\eta^2 R}{p\mu}\frac{\partial^2 T}{\partial x^2}; \qquad P_{zz} = p + \frac{3}{2}\frac{\eta^2 R}{p\mu}\frac{\partial^2 T}{\partial x^2}. \tag{13}$$

We shall show that the components P_{xx} and P_{zz} of the pressure tensor (13) for the case under consideration do not coincide, since

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = -\frac{\partial}{\partial x}\left(\frac{Q}{\chi}\right) \neq 0 \tag{14}$$

owing to the dependence of the coefficient χ on temperature according to the law

$$\chi = \chi_0 T^{n+1/2}, \tag{15}$$

where n is a constant, different for different gases and reflecting the well-studied effect of the influence of temperature on the effective diameter of molecular collisions ($^{7-18}$). Consequently, remembering that $Q = \text{const}$, and taking (15) into account, from (14) one can easily obtain:

$$\partial^2 T/\partial x^2 = [Q(n+1/2)/\chi_0 T^{n+3/2}]\partial T/\partial x. \tag{16}$$

Expressing $\partial T/\partial x$ in (16) through Q with the aid of (1), we finally obtain

$$\partial^2 T/\partial x^2 = -Q^2(n+1/2)/\chi_0^2 T^{2n+2}. \tag{17}$$

Consequently, according to (17) and (13):

$$P_{xx} \neq P_{zz}. \tag{18}$$

This result formally contradicts Pascal's law. However, this by no means implies its applicability under nonisothermal conditions. Experimental results obtained in the Laboratory of Surface Phenomena of the Institute of Physical Chemistry of the Academy of Sciences of the USSR show that such anisotropy is not observed for the case considered of a nonuniformly heated gas in a narrow slit.

Let us return to the pressure tensor (2) and (5). We shall show that $P^{(2)} = 0$ for the case in which $Q = \text{const}$. According to (9) and (17), we have

$$\omega_3 \frac{\partial^2 T}{\partial x^2} = -3Q^2(n+1/2)/\kappa_0^2 T^{2n+2}. \tag{19}$$

According to (1), (10), and also on the basis of the known dependence of the viscosity coefficient on temperature (see (7−18)) $\eta = \eta_0 T^{n+1/2}$, we have

$$\frac{\overline{\omega}_5}{T}\left(\frac{\partial T}{\partial x}\right)^2 = 3\frac{T}{\eta}\frac{\partial \eta}{\partial T}\frac{Q^2}{\kappa_0^2 T^{2n+2}}. \tag{20}$$

Further, on the basis of $\eta = \eta_0 T^{n+1/2}$, we have:

$$\partial \eta/\partial T = (n+1/2)\eta_0 T^{n-1/2} = (n+1/2)\eta/T. \tag{21}$$

From (20), taking (21) into account, we have:

$$\frac{\overline{\omega}_5}{T}\left(\frac{\partial T}{\partial x}\right)^2 = \frac{3Q^2(n+1/2)}{\kappa_0^2 T^{2n+2}}. \tag{22}$$

Substituting (19) and (22) into relation (5), we obtain: $P^{(2)} = 0$, and therefore,

$$P_{xx} = P_{zz}. \tag{23}$$

The result obtained permits us to assert that the investigation carried out by Maxwell was not rigorous, and that his neglect of terms proportional to the square of the temperature gradient in a number of cases leads to substantial errors, since the indicated terms, as rigorous analysis shows, turn out to be comparable in magnitude with the terms proportional to the second derivative of the temperature.

In conclusion, let us express the principal results in terms of the experimentally observed temperature gradient $\partial T/\partial x$. According to (17), (1), and (15),

$$\frac{\partial^2 T}{\partial x^2} = -\frac{(n+1/2)}{T}\left(\frac{\partial T}{\partial x}\right)^2. \tag{24}$$

According to Maxwell, the possible pressure anisotropy takes the form

$$|P_{xx} - P_{zz}| = \frac{3\eta^2 R(n+1/2)Q^2}{\rho\mu T\kappa^2} = \frac{3\eta^2 R(n+1/2)}{\rho\mu T}\left(\frac{\partial T}{\partial x}\right)^2. \tag{25}$$

It should be noted that, in the event that the condition $Q = \text{const}$ is not fulfilled, i.e., when thermal radiation is present in the volume of the gas, pressure anisotropy may occur. Indeed, the theoretical conclusion of the present work is valid only for the case of a constant flux.

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