RADIATION OF ELECTROMAGNETIC WAVES IN THE NONLINEAR INTERACTION OF SURFACE OSCILLATIONS IN A PLANE PLASMA LAYER

PHYSICS

1965

SovietRxiv

Abstract

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UDC 533.9

PHYSICS

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RADIATION OF ELECTROMAGNETIC WAVES IN THE NONLINEAR INTERACTION OF SURFACE OSCILLATIONS IN A PLANE PLASMA LAYER

(Presented by Academician E. K. Zavoiskii, March 16, 1965)

In a bounded electron plasma, along with volume electrostatic oscillations, there also exist surface oscillations, whose electric field decreases exponentially on both sides of the plasma boundary ($^{1-3}$). The frequency of volume electrostatic oscillations is equal to the electron plasma frequency ω_0, and the frequency of short-wavelength surface oscillations is equal to $\omega_0/\sqrt{2}$. The nonlinear interaction of volume oscillations leads to the appearance of electromagnetic radiation from the plasma ($^{4-6}$). The frequency of such radiation is equal to twice the frequency of the volume oscillations, i.e., $2\omega_0$. It is natural to expect that, in the nonlinear interaction of surface oscillations, radiation will arise whose frequency is equal to twice the frequency of these oscillations, i.e., $\omega_0\sqrt{2}$.

Radiation of electromagnetic waves at the frequency $2\omega_0$ has apparently been observed experimentally in experiments on turbulent heating of plasma in a toroidal system (^{7}). On the other hand, excitation of intense surface oscillations was observed in the same experiments. It is therefore of interest to estimate what contribution the nonlinear interaction of surface oscillations can make to the electromagnetic radiation.

We shall solve this problem for a plane plasma layer of thickness $2a$, assuming that the inequality $a \ll c/\omega_0$ is satisfied, where c is the speed of light and ω_0 is the electron plasma frequency. Such a condition is realized in many experiments, for example (^{7}). One may expect that our results will also be qualitatively applicable to a plasma cylinder. To simplify the exposition, we shall confine ourselves to determining the intensity of the radiation arising from the interaction of oscillations whose wave vectors k lie in the range $a^{-1} \ll k \ll r_D^{-1}$, where r_D is the electron Debye radius. In this case the thermal motion of the electrons may be neglected and the oscillations regarded as potential (^{8}). If the inequalities $a^{-1} \ll k \ll r_D^{-1}$ are valid, then the frequency of the surface oscillations is equal to $\omega_0/\sqrt{2}$ (^{8}).

Assuming that in the unperturbed state the electric and magnetic fields are zero and the electrons are at rest, one may write the following equation determining the perturbation of the electric field $\mathbf{E}$ (^{6}):

$$\Delta\mathbf{E} - \operatorname{grad}\operatorname{div}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi e^2 n_0}{mc^2}\mathbf{E} =$$

$$= \frac{4\pi e n_0}{c^2}(\mathbf{v}\nabla)\mathbf{v} + \frac{4\pi e^2 n_0}{mc^3}[\mathbf{v},\mathbf{H}] + \frac{1}{c^2}\frac{\partial}{\partial t}(\mathbf{v}\ \operatorname{div}\mathbf{E}), \tag{1}$$

where $\mathbf{v}$ is the perturbation of the electron velocity, $\mathbf{H}$ is the perturbation of the magnetic field, n_0 is the unperturbed plasma density ($n_0 = 0$ outside the layer), and the remaining notation is standard.

Since we are interested in the radiation of electromagnetic waves in the nonlinear interaction of surface oscillations, the perturbations $\mathbf{v}$, $\mathbf{E}$, and $\mathbf{H}$ in the surface oscillations should be substituted into the right-hand side of equation (1). Then from equation (1) it will be possible to find the field of the electromagnetic

waves. If we rewrite equation (1) in the form

$$\Delta\mathbf{E} - \operatorname{grad}\operatorname{div}\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi e^2 n_0}{mc^2}\mathbf{E} = \frac{4\pi}{c^2}\frac{\partial\mathbf{j}_{\mathrm{st}}}{\partial t},$$

where the nonlinear terms are denoted by $\frac{4\pi}{c^2}\frac{\partial\mathbf{j}_{\mathrm{st}}}{\partial t}$, it becomes clear that the problem reduces to determining the field of an electromagnetic wave for given external currents $\mathbf{j}_{\mathrm{st}}$. For a plane plasma layer, such a problem was solved in Ref. (^{6}).

Let us introduce a coordinate system whose x-axis is directed perpendicular to the layer, with the origin placed midway between the planes bounding the layer. We transform the quantities $\mathbf{E}$ and $\mathbf{j}_{\mathrm{st}}$ into a Fourier integral with respect to the coordinates y and z:

$$\mathbf{E} = (2\pi)^{-1}\int d\mathbf{k}\, e^{i\mathbf{kr}}\mathbf{E}_{\mathbf{k}}, \qquad \mathbf{j}_{\mathrm{st}} = (2\pi)^{-1}\int d\mathbf{k}\, e^{i\mathbf{kr}}\mathbf{j}_{\mathrm{st}\,\mathbf{k}},$$

where $\mathbf{k}$ is a two-dimensional vector in the (yz) plane. It is convenient to express the Fourier image of the external current $\mathbf{j}_{\mathrm{st}\,\mathbf{k}}$ in terms of the perturbation of the electrostatic potential φ in the surface oscillations. The function φ can be expanded in the eigenfunctions of the surface oscillations (^{1,2}):

$$\varphi = (2\pi)^{-1}\int d\mathbf{k}\, e^{i\mathbf{kr}}\left[\varphi_{\mathbf{k}}(x)e^{-i\frac{\omega_0}{\sqrt{2}}t} + \varphi_{-\mathbf{k}}(x)e^{i\frac{\omega_0}{\sqrt{2}}t}\right], \tag{2}$$

$$\varphi_{\mathbf{k}}(x) = \varphi_{\mathbf{k}}^{(s)}(x) + \varphi_{\mathbf{k}}^{(a)}(x),$$

$$\varphi_{\mathbf{k}}^{(s,a)}(x) = \varphi_{\mathbf{k}}^{(s,a)} \begin{cases} \exp[k(a-x)], & x > a, \\ \exp[k(x-a)] \pm \exp[-k(x+a)], & |x| < a, \\ \pm \exp[k(x+a)], & x < -a. \end{cases} \tag{3}$$

The eigenfunctions are written for the case $k \gg a^{-1}$; the indices s and a refer, respectively, to the even and odd eigenfunctions. The Fourier image of the external current is related to the quantities $\varphi_{\mathbf{k}}$ by the relations

$$\mathbf{j}_{\mathrm{st}\,\mathbf{k}}(x,t) = \mathbf{j}_{\mathbf{k}}^{(+)}(x) e^{-i\omega_0\sqrt{2}\,t} + \mathbf{j}_{\mathbf{k}}^{(-)}(x) e^{i\omega_0\sqrt{2}\,t}, \qquad \mathbf{j}_{\mathbf{k}}^{(-)} = \mathbf{j}_{-\mathbf{k}}^{(+)*}, \tag{4}$$

$$\mathbf{j}_{\mathbf{k}}^{(+)}(x) = -i\,\frac{e}{8\sqrt{2\pi^2 m\omega_0}} \int d\mathbf{k}'\,d\mathbf{k}''\,\delta(\mathbf{k}'+\mathbf{k}''-\mathbf{k}) \left(i\mathbf{k} + \vec{\xi}\frac{d}{dx}\right) \times \left[\frac{d\varphi_{\mathbf{k}'}}{dx}\frac{d\varphi_{\mathbf{k}''}}{dx} - (\mathbf{k}'\mathbf{k}'')\varphi_{\mathbf{k}'}\varphi_{\mathbf{k}''}\right], \tag{5}$$

where $\vec{\xi}$ is a unit vector in the direction of the x-axis. The vector $\mathbf{j}_{\mathbf{k}}^{(+)}$ is perpendicular to the vector $\mathbf{e} = k^{-1}[\mathbf{k}, \vec{\xi}]$. This means (^{6}) that the emitted electromagnetic waves have p-polarization. There is no emission of s-polarized waves.

It follows from formula (4) that the frequency of the electromagnetic radiation arising from the nonlinear interaction of surface oscillations is equal to $\omega_0\sqrt{2}$, i.e. the Fourier image of the electric field can be represented in the form

$$\mathbf{E}_{\mathbf{k}}(x,t) = \mathbf{E}_{\mathbf{k}}^{(+)}(x) e^{-i\omega_0\sqrt{2}\,t} + \mathbf{E}_{\mathbf{k}}^{(-)}(x) e^{i\omega_0\sqrt{2}\,t}, \qquad \mathbf{E}_{\mathbf{k}}^{(-)} = \mathbf{E}_{-\mathbf{k}}^{(+)*}.$$

Under the condition $a \ll c/\omega_0$, the Fourier image of the field of the electromagnetic wave in vacuum ($|x| > a$) is related to the external currents by the relations (^{6}):

$$\mathbf{E}_{\mathbf{k}}^{(+)}(x) = \frac{\mathbf{k}}{k} E_{k1}^{(+)}(x) + \vec{\xi} E_{k2}^{(+)}(x),$$

$$E_{k1}^{(+)}(x) = -\frac{2\sqrt{2}\pi}{\omega_0} \exp[\mp i\chi(x \pm a)] \int_{-x}^{a} dx' \left[\frac{\sqrt{2\omega_0^2 - k^2c^2}}{2kc}(\mathbf{k}\mathbf{j}_{\mathbf{k}}^{(+)}) \pm k(\vec{\xi}\mathbf{j}_{\mathbf{k}}^{(+)})\right],$$

$$E_{k2}^{(+)}(x) = \pm\frac{k}{\chi} E_{k1}^{(+)}(x), \tag{6}$$

where $\chi = \sqrt{2\omega_0^2/c^2 - k^2}$ is the component, perpendicular to the layer, of the wave vector of the electromagnetic wave outside the plasma. The upper signs in these formulas are taken for $x < -a$, the lower signs for $x > a$. Using relations (3), (5), and (6), we obtain:

$$E_{k1}^{(+)}(x) = \pm\frac{ike}{\pi m\omega_0^2}\exp[\mp i\chi(x \pm a)]\int d\mathbf{k}'\,d\mathbf{k}''\,\delta(\mathbf{k}'+\mathbf{k}''-\mathbf{k})\times$$

$$\times k'k''\left[\varphi_{k'}^{(s)}\varphi_{k''}^{(a)} + \varphi_{k'}^{(a)}\varphi_{k''}^{(s)}\right].$$

We have taken into account the inequalities $k', k'' \gg a^{-1}$, $k \leq \sqrt{2}\omega_0/c \ll a^{-1}$.

The time-averaged energy flux of the electromagnetic radiation in the interval $[\mathbf{k}, \mathbf{k} + d\mathbf{k}]$ is expressed by the formula

$$\mathbf{S}_k d\mathbf{k} = \mathbf{n}\,\frac{c}{4\pi L^2}\left(|E_k^{(+)}|^2 + |E_k^{(-)}|^2\right) = \mathbf{n}\,\frac{c}{4\pi L^2}\,\frac{k^2+\chi^2}{\chi^2}\left(|E_{k1}^{(+)}|^2 + |E_{-\mathbf{k}1}^{(+)}|^2\right),$$

where $\mathbf{n}$ is the unit vector in the direction of wave propagation, and L is the normalization length in the y and z directions.

Substituting here the expression found above for $E_{k1}^{(+)}$ and averaging over the phases of the perturbations of the potential $\varphi_k^{(s,a)}$, we obtain

$$\mathbf{S}_k = \mathbf{n}\,\frac{2\pi}{c}\left(\frac{ke}{\chi m\omega_0}\right)^2 I_k,$$

$$I_k = \int d\mathbf{k}'\,d\mathbf{k}''\,\delta(\mathbf{k}'+\mathbf{k}''-\mathbf{k})\,k'k''\left(W_{k'}^{(s)}W_{k''}^{(a)} + W_{-\mathbf{k}'}^{(s)}W_{-\mathbf{k}''}^{(a)}\right), \qquad (7)$$

where

$$W_k^{(s,a)} = 2\pi^{-1}L^{-2}k'|\varphi_k^{(s,a)}|^2$$

is the energy of the surface oscillations per unit area of the layer surface and per unit interval of wave vectors. Expression (7) is valid both for $x > a$ and for $x < -a$. We note that, under the condition $k', k'' \gg k$, the integral I_k does not depend on $\mathbf{k}$:

$$I_k \simeq I \equiv \int d\mathbf{k}'\,{k'}^2\left(W_{k'}^{(s)}W_{-\mathbf{k}'}^{(a)} + W_{-\mathbf{k}'}^{(s)}W_{k'}^{(a)}\right).$$

Let us calculate the energy S radiated from unit area of the layer surface into the half-space $x > a$. To do this, we integrate the projection of the vector $\mathbf{S}_k$

on the outward normal to the plane $x = a$ over $\mathbf{k}$ from $k = 0$ to $k = \sqrt{2}\omega_0/c$. The result of the integration has the form

$$S = \frac{16\pi^2}{3} \frac{e^2}{m^2c^3} I. \tag{8}$$

Exactly the same expression is obtained for the energy radiated into the half-space $x < -a$.

Let us estimate the magnitude of S, assuming that in the interval from $k = 0$ to $k \sim k_D = r_D^{-1}$ the energy of the surface and bulk oscillations is uniformly distributed over the degrees of freedom of the oscillatory motion (see ([9,10])). Under this condition the relation ([9])

$$W_k^{(s,a)} \simeq \frac{W}{2ak_D^3}$$

holds, where W is the total energy of the bulk and surface oscillations per unit area of the layer. Using this relation, we find that

$$S \simeq \frac{\pi^2}{3} \frac{c}{a} \left(\frac{\omega_0}{ck_D}\right)^2 W \frac{W}{amnc^2}. \tag{9}$$

For comparison, we write the formula that determines the intensity of the radiation associated with the nonlinear interaction of bulk electrostatic oscillations and having frequency $2\omega_0$ ([6]),

$$S \simeq \frac{16}{3} \frac{c}{a} \left(\frac{\omega_0}{ck_D}\right)^2 W \frac{W}{amnc^2}.$$

(this formula was obtained under the same assumptions as formula (9)). Thus, the radiation intensity at the frequency $\omega_0\sqrt{2}$ is comparable with the radiation intensity at the frequency $2\omega_0$.

Analogously to what was done above, one can determine the radiation intensity arising from the interaction of volume and surface oscillations. The frequencies of such radiation are equal to $\omega_0(1 \pm 1/\sqrt{2})$, and the intensity, to within inessential numerical factors, coincides with the radiation intensity at the frequencies $2\omega_0$ and $\omega_0\sqrt{2}$. Naturally, the thermal motion of the electrons, which we have not taken into account, leads to broadening of the lines in the radiation spectrum.

Summarizing the results of the present work and of work (6), one may assert that, in a bounded plasma with transverse size $a \lesssim c/\omega_0$, in the presence of intense volume and surface oscillations, the frequencies $2\omega_0$, $\omega_0\sqrt{2}$, and $\omega_0(1 \pm 1/\sqrt{2})$ will be present in the spectrum of the electromagnetic radiation of the plasma. Provided that the energy of the oscillations is distributed uniformly over the degrees of freedom, the radiation intensities at these frequencies are of the same order of magnitude. Our assertions can be checked experimentally on installations of type (7).

The author expresses his gratitude to A. A. Ivanov and L. I. Rudakov for discussion of the results of the work.

Received
16 III 1965

CITED LITERATURE

1. N. Herlofson, Ark. Fys., **3**, 247 (1951).
2. E. Astrom, Ark. Fys., **19**, 163 (1961).
3. P. E. Vandenplas, R. W. Gould, Physica, **28**, 537 (1962).
4. I. A. Akhiezer, I. A. Apanovich, N. P. Lininidze, JETP, **46**, 300 (1964).
5. R. E. Aamodt, W. E. Drummond, J. Nucl. Energy, C6, 147 (1964).
6. A. A. Ivanov, D. D. Ryutov, JETP, **48**, 684 (1965).
7. B. A. Demidov, N. I. Elagin et al., JETP, **48**, 454 (1965).
8. Yu. A. Romanov, JETP, **47**, 2119 (1964).
9. A. A. Ivanov, D. D. Rotov, JETP, **48**, 1366 (1965).
10. V. M. Dikasov, L. I. Rudakov, D. D. Ryutov, JETP, **48**, 913 (1965).

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