PHYSICS

1965

SovietRxiv

Abstract

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PHYSICS

V. V. VLADIMIROV

COLLISIONLESS CURRENT CONVECTION AND ITS DYNAMIC STABILIZATION

(Presented by Academician M. A. Leontovich on 10 XII 1964)

As is known ([1]), in a weakly ionized inhomogeneous high-pressure plasma situated in a constant longitudinal magnetic field, current convection may develop, associated with collisions of charged particles with neutrals, which create the phase shifts necessary for the development of the instability. An interesting modification of current convection may appear in an inhomogeneous low-pressure plasma situated in a constant magnetic field, when the collision frequency is smaller than all the other characteristic frequencies. In this case collisionless current convection may develop, in whose mechanism allowance for the inertia of the ions plays an important role. In analyzing this instability we start from the following equations, linearized with respect to perturbations: the kinetic equation for electrons

$$\frac{\partial f_e}{\partial t} + \vec{v}\vec{\nabla} f_e + \frac{e}{m_e}\vec{\nabla}\varphi \cdot \frac{\partial f_e}{\partial v} - [\vec{v}\vec{\omega}_{eH}]\frac{\partial f_e}{\partial v} = 0 \tag{1}$$

and the equations of hydrodynamics for cold ions

$$\frac{\partial n_i}{\partial t} + \operatorname{div} n\vec{v}_i = 0, \qquad \frac{d\vec{v}_i}{dt} = -\frac{e}{M}\vec{\nabla}\varphi + [\vec{v}_i\vec{\omega}_{iH}]. \tag{2}$$

In analyzing stability we shall use the quasiclassical approximation, choosing the perturbations of the stationary quantities in the form

$$A' = A' \exp(-i\omega t + ik_z z + ik_y y).$$

This approximation is valid when

$$|k_y| \gg |k_x| \gg |\chi| = \frac{1}{n_0}\left|\frac{dn_0}{dx}\right|.$$

The x-axis coincides with the direction of the density variation in the equilibrium state; the current and the magnetic field are directed along the z-axis.

Linearizing equations (1), (2) with respect to perturbations, it is easy to obtain expressions for the perturbed electron and ion densities:

$$n_e' = i\sqrt{\frac{\alpha}{2\pi}}\left[\frac{k_y c}{H}\frac{dn_0}{dx}\Phi + \frac{k_z e}{T_e}n_0(x)\Phi_1\right]\varphi';$$

$$\alpha = \frac{m_e}{2T_e}; \qquad \Phi = \frac{\pi}{k_z}W(\xi); \qquad \Phi_1 = -\frac{i}{k_z}\sqrt{\frac{\pi}{\alpha}}\left[1 + i\sqrt{\pi}\xi W(\xi)\right]; \tag{3}$$

$$\xi = \frac{\sqrt{\alpha}}{k_z}(\omega - k_z v_0); \qquad W(\xi) = e^{-\xi^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^\xi e^{t^2}\,dt\right).$$

Expression (3) was derived in the drift approximation $v_\perp = c[\mathbf{EH}]/H^2$, under the assumption that the unperturbed electron distribution is given by a Maxwellian function shifted by v_0 relative to the ions,

$$\frac{n_i'}{n_0\varphi'} = \frac{e}{M\omega}\left(-\frac{k_y\omega}{\omega_{iH}^2 - \omega^2} + \frac{k_z^2}{\omega} - \frac{k_y\chi\omega_{iH}}{\omega_{iH}^2 - \omega^2}\right). \tag{4}$$

In deriving (3), (4) the perturbations were assumed to be potential ones,

$$\mathbf{E}' = -\vec{\nabla}\varphi'.$$

In the case $\xi \to \infty$ $\left(\frac{\omega}{k_z} \gg v_{eT} = \sqrt{\frac{2T_e}{m_e}}\right)$,

$$W(\xi) = \frac{i}{\sqrt{\pi}}\frac{1}{\xi}\left(1 + \frac{1}{2\xi^2} + ...\right),$$

and for the perturbed electron density we easily obtain the usual hydrodynamic expression

$$\frac{n_e'}{n_0\varphi'} = -\frac{(k_{yc}/H)\chi}{\omega - k_{zv}0} - \frac{k_z^2 e}{m_e}\frac{1}{(\omega - k_{zv}0)^2}. \tag{5}$$

From the quasineutrality condition $n_i' = n_e'$, using (4), (5), we obtain the dispersion equation:

$$-\frac{(k_{yc}/H)\chi}{\omega - k_{zv}0} - \frac{k_z^2 e}{m_e}\frac{1}{(\omega - k_{zv}0)^2} = \frac{e}{M\omega}\left(-\frac{k_y\omega}{\omega_{iH}^2 - \omega^2} + \frac{k_z^2}{\omega} - \frac{k_y\chi\omega_{iH}}{\omega_{iH}^2 - \omega^2}\right). \tag{6}$$

Below we shall consider two cases: a) $\omega \gg \omega_{iH}$; b) $\omega \ll \omega_{iH}$. In both cases we study waves with $\omega \sim k_{zv}0$,

$$k_z^2 < \frac{k_y|\chi|}{\omega_{eH}}\omega. \tag{7}$$

In case a) condition (7) gives

$$k_z^2 < k_y|\chi|\frac{m_e}{M},$$

and in case b)

$$k_z^2 < \frac{k_y|\chi|}{\omega_{eH}} k_{zv}0.$$

These conditions are satisfied when the transverse wavelength is small and the longitudinal one is large, i.e., when the perturbed electric field is almost transverse. When condition (7) is satisfied, the second term on the left-hand side of equation (6) may be neglected, and the dispersion equation takes the form:

$$\text{a)}\quad \omega^2 + \frac{k_y}{\chi}\omega_{iH}\omega - \frac{k_y}{\chi}\omega_{iH}k_{zv}0 = 0; \tag{8}$$

$$\text{b)}\quad \omega^2 - k_{zv}0\omega - \frac{\chi}{k_y}\omega_{iH}k_{zv}0 = 0. \tag{9}$$

In case a)

$$\omega = -\frac{k_y}{2\chi}\omega_{iH} \pm \sqrt{\frac{\omega_{iH}^2}{4}\frac{k_y^2}{\chi^2} + \frac{k_y}{\chi}\omega_{iH}k_{zv}0}.$$

Taking into account that $\chi < 0$, we obtain the criterion for generation of instability

$$k_{zv}0 > \frac{\omega_{iH}}{4}\frac{k_y}{|\chi|}; \tag{8'}$$

$$\omega_{\text{cr}} = \frac{k_y}{2|\chi|}\omega_{iH} = 2k_{zv}0; \qquad \gamma = \sqrt{\frac{k_y}{|\chi|}\omega_{iH}k_{zv}0}.$$

The condition of ion nonmagnetization, which we used in deriving (8), is fulfilled when $k_y > 2|\chi|$.

In case b)

$$\omega = \frac{k_{zv}0}{2} \pm \sqrt{\frac{k_z^2 v_0^2}{4} + \frac{\chi}{k_y}\omega_{iH} k_{zv}0}.$$

The generation criterion has the form

$$k_z v_0 < 4\frac{|\chi|}{k_y}\,\omega_{iH}, \qquad \omega_{\rm cr} = \frac{k_z v_0}{2} = 2\frac{|\chi|}{k_y}\,\omega_{iH}, \qquad \gamma = \sqrt{\frac{|\chi|}{k_y}\,\omega_{iH} k_z v_0}. \tag{9'}$$

The condition of magnetization is satisfied for $k_y > 2|\chi|$.

In the considered variant of collisionless current convection,* waves are excited: in case a), in a definite interval of wave numbers k_z,

$$k_y\sqrt{\frac{m_e}{M}} > k_z > \frac{k_y}{|\chi|}\,\frac{\omega_{iH}}{4v_0}, \tag{10}$$

and in case b), for small k_z.

In both cases the growth rate of the instability is $\gamma \propto \sqrt{k_z v_0 H}$. If one sets $k_z \sim 1/L$, where L is the length of the discharge gap, then the frequency and growth rate of the instability in both cases coincide in order of magnitude with the reciprocal of the electron transit time through the discharge gap.

Since, in the instability considered, the allowance for ion inertia is quite important, it is of interest to consider dynamic stabilization of this instability by superposing a high-frequency current on the main current.

The equation describing the time variation of the perturbed density $n' = n'(t)\exp(ik_z z + ik_y y)$ in the unstable regime, according to the original equations (1), (2), with the indicated approximations taken into account, has the form:

$$\frac{d^2 n'}{dt^2} - \varepsilon\omega_{iH} k_z v_0 n' = 0, \tag{11}$$

where

$$\varepsilon = \begin{cases} \dfrac{k_y}{|\chi|}, & \text{in case a)}, \\ \dfrac{|\chi|}{k_y}, & \text{in case b)}, \end{cases} \qquad v_0 = v_{0c} + \tilde{v}_0, \qquad \tilde{v}_0 = \tilde{v}_0 \cos\omega_0 t.$$

If we introduce the notation:

$$a = -\frac{4}{\omega_0^2}\varepsilon\omega_{iH}k_z v_{0c}; \qquad q = -\frac{a}{2}\frac{\tilde{v}_0}{v_{0c}}; \qquad \frac{\omega_0 t}{2} = \tau,$$

then equation (11) takes the form of the Mathieu equation

$$\frac{d^2 n'}{d\tau^2} + (a - 2q\cos 2\tau)n' = 0,$$

whose stability regions have been investigated in (3), and, as applied to the dynamic stabilization of a plasma column, in (4).

Determination of the stability region of the solutions of equation (11) can be simplified if it is reduced to the equation of motion of an inverted pendulum with a vibrating suspension point (5). (A detailed analysis of the dynamic stabilization of the motion of an inverted pendulum was carried out by P. L. Kapitsa (6). As applied to dynamic stabilization of a plasma column, this method was used by S. M. Osovets (7).) The equation has the form:

$$\frac{d^2 n'}{dt^2} = \left(gL^{-1} - aL^{-1}\omega_0^2 \sin\omega_0 t\right) n' \tag{12}$$

(it is easily obtained from (11) after the substitution $\omega_0 t = \pi/2 + \omega_0 t$), where

$$g = \varepsilon\omega_{iH}v_{0c}; \qquad L^{-1} = k_z; \qquad a = \varepsilon\omega_{iH}\tilde{v}_0/\omega_0^2.$$

* **Note added in proof.** As became known to the author after this work had been sent to press, the case $k_z v_z \ll \omega \ll \omega_{iH}$ of the instability considered was discussed by A. B. Mikhailovskii (8) as applied to the experiment of Nezlin (2).

The stability condition for the solutions (12) for $a/L \ll 1$ has the form [5)]

$$a^2\omega_0^2 \geq 2gL, \tag{13}$$

whence

$$\frac{\omega_0^2}{\varepsilon\omega_{iH}k_z v_{0c}} \gg \frac{\tilde{v}_0}{v_{0c}} > \frac{\omega_0}{\varepsilon\omega_{iH}}\sqrt{\frac{2\varepsilon\omega_{iH}}{k_z v_{0c}}}. \tag{14}$$

From (14) it follows that

$$\omega_0 \gg \sqrt{2\varepsilon\omega_{iH}k_z v_{0c}} \simeq \gamma; \qquad \frac{\tilde{v}_0}{v_{0c}} > \frac{\omega_0}{\omega_{iH}}\sqrt{\frac{2\omega_{iH}}{\varepsilon k_z v_{0c}}} \simeq \frac{\omega_0}{\gamma},$$

where γ is the instability increment.

Thus, by superposing on the main current a high-frequency current (with a frequency greater than the inverse time of flight of an electron across the discharge gap) of sufficient amplitude ($\tilde{j} > j_c$), one can stabilize short-wavelength perturbations.

In conclusion I express my deep gratitude to B. B. Kadomtsev and A. V. Timofeev, who made a number of valuable comments.

Received
24 XI 1964

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Note: Figure translations are in progress. See original paper for figures.