CONSTRUCTION OF INVERSE OPERATORS IN THE THEORY OF AUTOMATIC CONTROL

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Abstract

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CONSTRUCTION OF INVERSE OPERATORS IN THE THEORY OF AUTOMATIC CONTROL

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The solution of certain problems of automatic control (in particular, the problem of ensuring invariance in nonlinear control systems ([1–4])) reduces to the construction of an operator that is a right inverse for the operator of the controlled plant.

We shall state the formulation of the problem. Let the vector-function $\mu(t)$ be composed of the coordinates of the controlling elements, and let the vector-function $\varphi(t)$ be composed of the coordinates of the controlled processes, and let the equation of the controlled plant have the form*

$$\varphi(t) = A[\mu(t)], \tag{1}$$

where t is time. The operator A will be called the **operator of the controlled plant**. It is assumed that (1) is a system of equations (algebraic, nonlinear differential, nonstationary, integral, functional) that can be solved on a mathematical model ([5]) consisting of a finite number of blocks of the following types:

$$B_1\text{—integration: } y(t) = c + \int_0^t x(t)\,dt;$$

$$B_2\text{—differentiation: } y(t) = dx/dt;$$

$$B_3\text{—shift: } y(t) = x[t - \tau(t)];$$

B_4—functions of one or several variables:

$$y = f(x_1, \dots, x_n). \tag{2}$$

The functions $x(t), \tau(t), x_1(t), \dots, x_n(t)$ will be called **processes at the inputs of the blocks**, and the functions $y(t)$ **−processes at the outputs of the blocks**. The process at each input d of a block D is either identical with the process at the output of some block C, or coincides with one of the components of the vector-function $\mu(t)$. In the first case we shall say that a connection goes from the output of block C to input d**; in the second case, input** d **will be called an** input of the construction**.

A scheme on which the blocks of the mathematical model and the connections joining them are indicated will be called a **construction** realizing the operator, or simply a construction of the operator. The **outputs of the construction** are the outputs of blocks whose processes are components of the vector $\varphi(t)$.

The problem consists in the following: given a construction of the operator of the controlled plant A, find a construction of the right inverse*** operator A_r^{-1}, using only blocks of the types listed above.

* For simplicity of exposition, we do not take into account here the dependence of $\varphi(t)$ on external disturbances (loads). Everything that follows is easily extended to this case as well.

** Only one connection may arrive at each input of a block, but several connections may go from an output of a block. The direction of a connection is taken to be from output to input.

*** A right inverse for the operator A is, as is known ([8]), an operator A_r^{-1} such that the operator AA_r^{-1} is the identity.

In somewhat different terminology, an analogous problem was formulated in [6]. In the present paper a solution of this problem is proposed, close to the solution proposed in [7] for the linear case.

Consider an ordered sequence, made up of blocks and connections, of an operator construction
$(C_1;\ s_{1,2};\ C_2;\ s_{2,3}; \dots;\ s_{m-1,m};\ C_m)$, where C_i are blocks of the types considered above; $s_{i,i+1}$ is a connection going from the output of block C_i to one of the inputs of block C_{i+1}. If one of the inputs of block C_1 is an input of the construction, and the output of block C_m is an output of the construction, then such a sequence will be called a **through channel**. A set of nonintersecting (i.e., having no common blocks) through channels will be called a **complete right-hand path** if the number of channels is equal to the number of outputs of the construction.

Consider a block C_k lying on a through channel, and let $x_k(t)$ be the process at that input of the block to which the connection $s_{k-1,k}$ comes, and $y(t)$ the process at the output of block C_k. Fixing the processes at the other inputs of the block (if there are any), we obtain the realization of a certain operator $y(t) = Qx_k(t)$, depending both on parameters and on the processes at the other inputs of the block. The block realizing the operator Q_r^{-1} will be called the right inverse block with respect to block C_k.*

We shall call a construction **right-inverted** if, in some complete right-hand path set of through channels, all blocks are replaced by right inverse ones, the connections by inversely directed ones, the branches by inverse branches, while the remaining blocks and connections are left unchanged. If in the construction one can select such a complete right-hand path set of through channels that all blocks lying on these channels have right inverses, then we shall say that a right-inverted construction exists.

Theorem 1. *If the right inverse operator A_r^{-1} and a right-inverted construction exist, then the right-inverted construction realizes the operator A_r^{-1}.*

It follows from this theorem that, in order to construct a construction of the right inverse operator A_r^{-1}, it suffices, in a construction realizing the operator A, to select a complete right-hand path set of through channels and to replace the blocks entering into it by right inverse ones, and the connections by inversely directed ones.

Consider some construction, and in it an arbitrary block C, from whose output the connections $s_1, \dots, s_l$ go. Take an identity block** D, not contained in the construction, connect its input to the output of block C by some connection, and disconnect the connections $s_1, \dots, s_l$ from block C and connect them to the output of block D. We obtain a new construction, which we shall call **equivalent** to the former one. It is obvious that equivalent constructions realize one and the same operator. In what follows all equivalent constructions will be identified.

Let some set Φ of processes at the outputs of a construction of the operator A be fixed (a set of n-dimensional vector functions $\varphi(t)$). A set of blocks will be called **ordinary** with respect to Φ if no construction from the blocks of this set (with the possible addition only of identity blocks) with a number of inputs less than n can realize an operator whose range contains the whole set Φ.

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* For example, for block B_2 the right inverse will be block B_1. If, solving equation (2) with respect to x_k, we obtain a multivalued, generally speaking, function
$x_k = \xi(y, x_1, \dots, x_{k-1}, x_{k+1}, \dots, x_n)$, then the block realizing one of the branches of this function is right inverse with respect to block B_4.

** We shall call a block an identity block if the process at the input coincides with the process at the output.

Theorem 2. *Any construction of an operator with set of values Φ, made up of a set of blocks ordinary with respect to Φ, contains a complete right-hand set of through channels.*

Theorem 2 follows from the following lemma, from whose proof one can also obtain an algorithm for finding a complete right-hand set of through channels.

By cutting some of the connections, a construction K can be divided into mutually unconnected parts K_i. By the inputs of a part K_i we shall mean the inputs of the construction that have ended up in K_i, as well as the inputs of blocks from K_i to which the cut connections go.

Lemma. *Let the maximum number of nonintersecting through channels of a construction be equal to k. Then one can single out a part of the construction having only k inputs and including all n outputs of the construction.*

Proof. Color all blocks and connections of k nonintersecting through channels red, and the remaining $(n-k)$ outputs yellow; let the yellow color spread from the yellow outputs only against the direction of the connections and only along connections and blocks not colored red. If even a particle of yellow paint were to reach one of the inputs of the construction, its trajectory would be one more through channel not intersecting the other k; it remains to assume that only a zone not containing inputs of the construction will be colored yellow. Suppose that from the outputs of the blocks $C_1, C_2, \dots, C_{j-1}$ of the through channel colored red

$$(C_1;\ s_{1,2};\ C_2;\ s_{2,3};\ \dots;\ s_{m-1,m};\ C_m)$$

no yellow connections emerge, while from the output of the block C_j at least one yellow connection goes. Then the set

$$(C_j;\ s_{j,j+1};\ \dots;\ C_m)$$

will be called the ending of the through channel. (If no yellow connections emerge from the output of any of the blocks of the channel, then the output of the block C_m will be regarded as the ending of the through channel.)

It may be assumed that the first block of the ending of a through channel (the block C_j) has one input. (If this is not so, then after the block C_j include an identity block D in the same way as was done in defining an equivalent construction. Then the first block of the ending of the through channel will be the block D with one input.)

Uncolored connections cannot arrive at the inputs of yellow blocks, since such connections would themselves become colored. If among the uncolored connections there are also no connections that arrive at the inputs of blocks entering into the endings of through channels, then the red-yellow zone has only k inputs (namely, the inputs of the first blocks of the endings) and is the required part of the construction.

In the opposite case, replace the ending to the input of whose block an uncolored connection arrives by the trajectory of the paint particle that colored the yellow connection issuing from the first block of the ending; correspondingly, recolor the new ending red and the former ending yellow. Then the aforementioned uncolored connection approaching the block of the former ending will be colored yellow; after it the block from whose output it goes will be colored (if this block is not colored), and so on. The red-yellow zone will increase, but, for the previous reason, the yellow color will not reach the inputs of the construction. Since the increase of the red-yellow zone is bounded because the number of blocks and connections of the construction is finite, repeating the operations described several times, we arrive at a situation in which uncolored connections do not approach the endings of through channels. The lemma is proved.

Everything set forth above carries over, with slight changes, to the case of constructing a left inverse-operator construction and an inverse-operator construction. In particular, to construct an inverse-operator construction it is necessary to take a set of nonintersecting through channels containing not only all outputs but also all inputs (the number of inputs of the construction must coincide with the number of outputs), and the blocks are replaced by inverse ones, and not by right inverses.

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Note: Figure translations are in progress. See original paper for figures.