QUANTUM DEPOLARIZATION OF ELECTRONS

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Abstract

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QUANTUM DEPOLARIZATION OF ELECTRONS

IN A MAGNETIC FIELD

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As was shown in ([1]), radiation during motion in a magnetic field leads to the polarization of electrons and positrons. Although the probability of radiation with a change in the direction of the spin is very small in comparison with the total probability of radiation, high-energy electrons and positrons, after prolonged rotation in modern storage rings, may become strongly polarized. The polarization time in a homogeneous field is equal to

$$\tau_{\text{pol}}^{-1} = \frac{5\sqrt{3}}{8}\alpha m \left(\frac{\varepsilon}{m}\right)^2 \left(\frac{H}{H_0}\right)^3, \qquad \alpha = \frac{1}{137}; \tag{1}$$

here $H_0 = \dfrac{m^2}{e} = 4.4 \cdot 10^{13}$ oersted. In a storage ring with energy $E = 6$ Bev and a field on the orbit $H = 8 \cdot 10^3$ oersted, $\tau_{\text{pol}} = 190$ sec.

In order to preserve the polarization of particles in a storage ring that arises in this way, it is necessary to choose the particle energy ε so that depolarizing resonances, caused by the radial and azimuthal components of the magnetic field on the particle trajectory, do not arise in the system. More precisely, the resonance condition

$$G\frac{\varepsilon}{m} = k + lQ_z + mQ_R + nQ_x, \qquad k,\ l,\ m,\ n \text{ integers}, \tag{2}$$

must be satisfied only for as large as possible values of l, m, n. Here $G = g - 2 = \alpha/2\pi$ is the anomalous part of the electron g-factor; $Q_{z,R,x}$ is the number of oscillations in the orbit for, respectively, vertical z-, radial R-, and phase x-oscillations. The terms lQ_z and mQ_R arise because of the presence of the terms z' and r^m ($r = R - R_0$, R_0 is the equilibrium radius) in the fields H_R, H_x; the term nQ_x takes into account synchrotron oscillations of the energy and the corrections to the frequencies $Q_{z,R}$ associated with these oscillations. For the correct choice of the energy ε, a detailed analysis of the given storage ring is required, taking into account the specific nonlinearity of the magnetic field, etc.

Suppose that the energy ε can be chosen so that the effects of depolarization under the action of the resonances (2) are negligible. It turns out that at sufficiently high energies there arises yet another possibility of depolarization, caused by the quantum character of the radiation. It also occurs only in the presence of perturbing fields H_R and H_x, but satisfaction of the resonance conditions (2) is now not necessary. The effect is caused by the fact that the energy jumps associated with the quantum character of radiation, when expanded in a Fourier integral, contain, in particular, harmonics that produce the resonance (2). Therefore the phenomenon here too is based on resonance, with the unpleasant feature that it cannot be avoided by the choice of ε. We would like to emphasize here that we have been unable to discover any other classical or quantum effects leading to depolarization of the beam, so that, by eliminating the most dangerous harmonics of the field perturbations, one can sharply reduce the depolarization effects. This also applies to the effect considered below.

The consideration of quantum depolarization can be carried out in an approximation in which the trajectory of the electron is regarded as classical, while the energy

the particle undergoes jumps in each act of emission, so that

$$\frac{\overline{d(\Delta\varepsilon)^2}}{dt} = \frac{55\sqrt{3}}{48}\frac{1}{R}\left(\frac{\varepsilon}{m}\right)^3 \overline{W}, \qquad \overline{W} = \frac{2}{3}\left(\frac{\varepsilon}{m}\right)^4 \frac{r_0 m}{R^2}; \tag{3}$$

here $\overline{W}$ is the classical radiation power of the electron, and r_0 is the classical electron radius.

The equations for the electron spin 4-vector S^i, taking into account the perturbing fields H_R and H_x, have the form (2)

$$\frac{dS^1}{d\tau} = G\frac{eH}{m}\left(\frac{\varepsilon}{m}\right)^2 S^2 - \left(1 + G\frac{\varepsilon^2}{m^2}\right)\frac{eH_R}{m}S^3; \tag{4}$$

$$\frac{dS^2}{d\tau} = -G\frac{eH}{m}S^1 + (1+G)\frac{eH_x}{m}S^3; \tag{5}$$

$$\frac{dS^3}{d\tau} = (1+G)\frac{eH_R}{m}S^1 + \left[G\frac{eH}{m}\frac{p}{m}\frac{dz}{dt} - (1+G)\frac{eH_x}{m}\right]S^2; \tag{6}$$

$$\frac{dS^4}{d\tau} = G\frac{eH}{m}\frac{\varepsilon p}{m^2}S^2 - G\frac{eH_R}{m}\frac{\varepsilon p}{m^2}S^3, \tag{7}$$

where $\tau = tm/\varepsilon$, and H is the equilibrium field. It is not difficult to see that neither spatial nor temporal variations of the field H lead to depolarization, which, of course, is evident in advance. Terms with H_R and H_x can give a resonant rotation of the spin, but we shall assume that condition (2) is not satisfied. Since the effect under consideration depends only indirectly, for the most part, on the type of beam focusing in the storage ring, we shall consider, for simplicity, an azimuthally symmetric weakly focusing field, in which the k-th harmonic of the perturbation acts,

$$H_R = h\cos\left[k\frac{eH}{m}(\tau-\tau_0)\right]$$

(at $z=0$), so that the field on the actual vertically perturbed orbit $z=z_H$ can be written in the form

$$H_R = -\frac{k^2 h}{n-k^2}\cos\left[k\frac{eH}{m}(\tau-\tau_0)\right]; \tag{8}$$

$$z_H = z_k\cos\left[k\frac{eH}{m}(\tau-\tau_0)\right], \qquad z_k = \frac{R}{H}\frac{h}{n-k^2}. \tag{9}$$

Since r-oscillations, in the approximation linear in h, do not contribute to the effect, we shall put $r=0$. The calculation of the effects caused by free z-oscillations in an inhomogeneous field, as well as by the field H_x, can be carried out in an analogous way; the corresponding results are given at the end of the paper.

We shall write the solution of equations (4)–(7) in first approximation in h in the rest frame of the particle. To transform to this system it is necessary to make a Lorentz transformation and an ordinary rotation through a small angle

$$\vartheta_z = \frac{m}{p}\frac{dz}{dt} = -\frac{h}{H}\frac{k}{n-k^2}\sin\left[k\frac{eH}{m}(\tau-\tau_0)\right]. \tag{10}$$

In this system (which we denote by the subscript c), $S_c^4=0$,

$$(S_c^1)^2+(S_c^2)^2 = S_\rho^2 = \sigma_\rho^2+\sigma_\rho\sigma_z\frac{h}{H}\frac{Gk}{n-k^2}F, \tag{11}$$

$$(S_c^3)^2 = S_z^2 = \sigma_z^2-\sigma_\rho\sigma_z\frac{h}{H}\frac{Gk}{n-k^2}F; \tag{12}$$

$$F = \frac{1+k\varepsilon/m}{(k-G\varepsilon/m)}\sin\left\{\frac{eH}{m}\left[\left(k-G\frac{\varepsilon}{m}\right)\tau-k\tau_0\right]\right\} - \frac{1-k\varepsilon/m}{(k+G\varepsilon/m)}\sin\left\{\frac{eH}{m}\left[\left(k+G\frac{\varepsilon}{m}\right)\tau-k\tau_0\right]\right\}, \tag{13}$$

where σ_z, σ_ρ are constants for constant particle energy, with always

$$\sigma_\rho^2 + \sigma_z^2 = 1. \tag{14}$$

The quantities S_z and S_ρ have the meaning of instantaneous projections of the spin onto the z-axis and onto the plane perpendicular to it in the electron rest frame; σ_z and σ_ρ have the meaning of the mean values about which the small oscillations S_z and S_ρ occur.

Because the relative probability of a spin flip at the moment of emission is negligibly small in comparison with the probability of emission without a spin flip, the magnitudes of the true spin projections S_z and S_ρ do not change at the moment of emission. However, at the moment of emission there occurs a jump-like change of F in formulas (11), (12), proportional to the jump $\Delta\varepsilon$. Since S_z and S_ρ do not change, this leads to a jump-like change of the averages σ_z and σ_ρ. Compensation of radiation losses by the accelerating system of the storage ring subsequently returns the value of F (more precisely, the values of the amplitudes in (13)) to its initial value; however, the totality of such random jumps leads to stochastic buildup of σ_z and σ_ρ, and consequently to buildup of S_z and S_ρ.

For sufficiently high energies, $G\varepsilon/m \gg 1$, for which the effect under consideration is of interest, the main contribution to the buildup process is made by jumps of the denominator in the first term of (13). In this case the principal role is played by harmonics k for which $G\varepsilon/m \sim k$. This gives the following change of the angle ϑ between the spin direction and the z axis:

$$\frac{1}{\tau_{\text{depol}}} \approx \frac{d\overline{\vartheta^2}}{dt} \approx \frac{1}{8}\sum_k \left(\frac{z_k}{R}\right)^2 \left(\frac{kG\varepsilon/m}{k - G\varepsilon/m}\right)^4 \frac{1}{\varepsilon^2}\frac{d\overline{(\Delta\varepsilon)^2}}{dt} =$$

$$= \sum_k \frac{55}{192\sqrt{3}} \left(\frac{kG\varepsilon/m}{k - G\varepsilon/m}\right)^4 \frac{r_0}{mR^3} \left(\frac{z_k}{R}\right)^2 \left(\frac{\varepsilon}{m}\right)^5; \tag{15}$$

here z_k is the amplitude of the forced z-oscillations excited by the perturbation H_R.

It is seen from (15) that the effect depends extremely strongly on the particle energy and depends strongly on the number of the nearest resonant harmonic k, on the distance to resonance $|k - G\varepsilon/m|$, and on the magnitude of z_k. Let us estimate the effect for reasonable values of the parameters: $E = 6$ BeV, $H = 8 \cdot 10^3$ oersted, $R = 3 \cdot 10^3$ cm, $|k - G\varepsilon/m| \approx 1/2$ for $k = 14, 15$, $z_k = 0.1$ cm. Then the characteristic depolarization time (change of the angle ϑ by unity) is $\tau_{\text{depol}} = 25$ sec. This is an order of magnitude less than τ_{pol}. Thus, in this case the beam is certainly not polarized. This means that special measures must be taken to preserve the beam polarization. Nevertheless, we would like once more to note the remarkable stability of the polarization.

We also give the formulas for S_ρ and S_z with allowance for the perturbation H_x and for the presence of free z-oscillations (these effects, generally speaking, make a small contribution in comparison with those considered above):

$$S^2_{\rho,z} = \sigma^2_{\rho,z} \pm \sigma_\rho \sigma_z \frac{h_{xk}}{H} \left\{ \frac{1+G}{k+G\varepsilon/m} \sin\left[\frac{eH}{m}\left(k+G\frac{\varepsilon}{m}\right)\tau - k\tau_1\right] - \right.$$

$$\left. -\frac{1+G}{k-G\varepsilon/m} \sin\left[\frac{eH}{m}\left(k-G\frac{\varepsilon}{m}\right)\tau - k\tau_1\right]\right\}, \qquad H_x = h_{xk}\sin\left[k\frac{eH}{m}(\tau-\tau_1)\right]; \tag{16}$$

$$S^2_{\rho,z} = \sigma^2_{\rho,z} \pm \sigma_\rho \sigma_z \frac{z_{\max}}{R} G \left\{ \frac{1+\frac{\varepsilon}{m}\sqrt{n}}{G\frac{\varepsilon}{m}\frac{1}{\sqrt{n}}-1} \sin\left[\frac{eH}{m}\left(G\frac{\varepsilon}{m}-\sqrt{n}\right)\tau + \sqrt{n}\tau_2\right] + \right.$$

$$\left. + \frac{\frac{\varepsilon}{m}\sqrt{n}-1}{G\frac{\varepsilon}{m}\frac{1}{\sqrt{n}}+1} \sin\left[\frac{eH}{m}\left(G\frac{\varepsilon}{m}+\sqrt{n}\right)\tau - \sqrt{n}\tau_2\right]\right\}. \tag{17}$$

As was to be expected, consideration of linear z-oscillations leads to a dependence of the effect on the difference $G\varepsilon/m - Q_z$, $Q_z = \sqrt{n}$ (cf. formula (2) for $n = 1$).

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.