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Abstract

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ON THE ONE-DIMENSIONAL INSTABILITY OF DETONATION

R. M. Zaidel and Ya. B. Zel' dovich ([1]) showed that in those cases where detonation in the Zel'dovich–Neumann model is stable with respect to curvature of the flame front (for example, in narrow tubes), it may be unstable with respect to one-dimensional perturbations. Suppose that the ignition-delay period in a plane Chapman–Jouguet detonation is, for a random reason, shortened. This is equivalent to an increase in the velocity of propagation of the flame through the gas compressed by the shock wave preceding the combustion zone. An increase in the flame velocity leads to the appearance of shock waves propagating on both sides of the combustion zone: through the compressed gas and through the combustion products. (In the general case the shock waves are separated by a rarefaction wave, which may be neglected if the perturbation is small.) The shock wave traveling through the compressed gas raises its temperature and shortens the ignition-delay period; therefore the initial perturbation grows. An analogous argument may also be carried out for the case of an increase in the ignition-delay period, corresponding to a decrease in the flame velocity and causing a rarefaction wave in the unburned gas. This wave lowers the gas temperature and increases the ignition delay.

In work ([1]) this qualitative scheme, treated quantitatively, leads to an instability criterion coinciding with the criterion for detonation instability with respect to curvatures of the combustion zone, derived by R. M. Zaidel ([2]):

$$\frac{E}{RT} > \frac{(h+3)(h+1+\sqrt{h+1})}{2(h+2)}. \quad (1)$$

Here E is the activation energy of the ignition reaction, R is the gas constant, T is the temperature of the compressed but unburned gas, $h = (\gamma+1)/(\gamma-1)$, and γ is the ratio of specific heats.

Below, the problem of the one-dimensional instability of detonation is considered anew and an instability criterion is derived that does not coincide with (1).

In a Chapman–Jouguet detonation the ignition zone is at rest with respect to the shock front. Therefore the velocity of flame propagation is equal in magnitude and opposite in sign to the flow velocity of the unburned gas in the coordinate system connected with the front,

$$u = -u_{\mathrm{g}}. \tag{2}$$

The distance from the shock front to the ignition zone is equal to

$$\lambda = u\tau,$$

where τ is the ignition delay.

In the unperturbed wave, over an arbitrary time interval t, a volume of gas, referred to unit surface area of the front, burns equal to

$$\dot{V} = ut.$$

If the ignition delay time is shortened for a random reason by an interval t by $\delta\tau$, the flame front will approach the shock front by a distance

$$\Delta\lambda = u\,\delta\tau,$$

and the volume of gas burned during this same time interval t will become equal to

$$V' = ut + u\,\delta\tau.$$

The ratio of the volumes of gas burned in one and the same time is equal to the ratio of the flame-propagation velocities

$$\frac{V'}{V} = \frac{u'}{u} = 1 + \frac{\delta\tau}{t}.$$

Since the time interval t is arbitrary, we shall take it equal to τ. Then, taking the signs into account, one may write

$$\Delta u = \frac{u' - u}{u} = -\frac{\delta\tau}{\tau}. \tag{3}$$

In applying relation (3), it should be borne in mind that Δu is the dimensionless increase in the flame velocity averaged over the time interval τ. If in the next time interval, equal to τ, the delay period is shortened further by $\delta\tau$, then the flame front will advance toward the shock front by another $u\,\delta\tau$. In other words, in order for the new flame velocity u' to be maintained, the ignition delay period must decrease continuously. If, however, after having decreased once by the amount $\delta\tau$, it remains equal to $\tau - \delta\tau$, the combustion zone will be established at a new distance $\lambda - u\,\delta\tau$ from the shock front. The flame will be at rest relative to the shock front.

The dimensionless pressure jump in the shock wave traveling through the unburned gas, caused by a small increase in the flame velocity, is determined, according to (3), by the expression

$$\Delta p = \frac{p' - p}{p} = -\frac{c}{c + c_\text{r}}\, qM\Delta u. \tag{4}$$

Here p' is the pressure in the shock wave; p is the pressure in the gas through which the shock wave travels; c and c_r are the speeds of sound in the unburned and burned gases, respectively; q is the ratio of the heat effect of combustion to the internal energy of the unburned gas; M is the ratio of the flame velocity to the speed of sound in the unburned gas.

The subsequent fate of the initial perturbation ($\delta\tau$ in time and $u\,\delta\tau$ in length) depends on what the feedback effect of the shock wave (4) will be on the ignition delay period and, correspondingly, on the flame velocity determined by expression (3). If, in a time not exceeding τ, the shock wave shortens the delay period by an amount exceeding $\delta\tau$, the initial perturbation will grow. If, however, the shock wave shortens the ignition delay by less than $\delta\tau$, the initial perturbation will decrease. Consequently, to determine the conditions for growth of the initial perturbation, it is necessary to find the dependence of the change in the ignition delay period (the dimensionless change in the flame velocity) on the dimensionless pressure jump in the shock wave.

The chemical-reaction time depends on the temperature as

$$\tau = A \exp \frac{E}{RT}. \tag{5}$$

Therefore

$$\tau + \delta\tau = A \exp \frac{E}{R(T + \delta T)}$$

or

$$\frac{\delta\tau}{\tau} = -\frac{E\,\delta T}{RT^2}. \tag{6}$$

Assuming the compression in the shock wave to be adiabatic, we obtain

$$-\frac{\delta\tau}{\tau} = \Delta u = \frac{E}{RT}\left[1 - \left(\frac{p'}{p}\right)^{(\gamma-1)/\gamma}\right]. \tag{7}$$

Expanding (7) in a series and restricting ourselves to the first approximation, we find

$$\Delta u = \frac{\gamma - 1}{\gamma} \frac{E}{RT} \frac{\delta p}{p} = \frac{\gamma - 1}{\gamma} \frac{E}{RT} \Delta p. \tag{8}$$

Using the instability condition ((3), p. 229, formula 31.2)

$$\left. \frac{d\Delta p}{d\Delta u} \right|_4 > \left. \frac{d\Delta p}{d\Delta u} \right|_8,$$

we arrive at the criterion for the onset of one-dimensional detonation instability in the form

$$\frac{\gamma - 1}{\gamma} \frac{E}{RT} \frac{1}{1 + c_r/c} qM > 1. \tag{9}$$

Criterion (9), in contrast to (1), contains, in particular, the heat effect of the reaction. This is natural, since, for given kinetic properties of the mixture, the ignition-delay period changes the more strongly, the more intense the shock wave caused by the specified change in flame velocity.

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CITED LITERATURE

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3. K. I. Shchelkin, Ya. K. Troshin, *Gas Dynamics of Combustion*, Publishing House of the Academy of Sciences of the USSR, 1963, p. 221.

Note: Figure translations are in progress. See original paper for figures.