On the book by Yu. A. Mitropolsky "Problems of the Asymptotic Theory of Non-stationary Oscillations"

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Date: 1965-01-01T00:00:00+00:00

Abstract

Full Text

Preamble

DIFFERENTIAL EQUATIONS, January 1965, Vol. I, No. 1 *Mitropolsky, Yu. A. Problems of the Asymptotic Theory of Non-stationary Vibrations.* "*Nauka*" *Publishing House, Moscow, 1964, 424 pp.*

Numerous problems associated with the calculation of non-stationary oscillatory processes necessitate the solution of nonlinear differential equations and systems of nonlinear differential equations with variable coefficients. Such problems include, for example, the classical problem of passing through resonance, which is inevitably encountered when designing crankshafts, turbine rotors, transmissions, turbine blades, propellers, centrifuges, and other mechanical components. This field also encompasses critical tasks related to the study of oscillatory phenomena in the calculation of rocket trajectories and satellite orbits, as well as problems associated with the design of high-power particle accelerators.

Building upon the fundamental ideas of the Krylov-Bogolyubov asymptotic methods, Yu. A. Mitropolsky developed an effective method for investigating these problems as early as 1955. The algorithm he developed allowed for the construction of asymptotic approximate solutions for nonlinear differential equations with variable coefficients, describing non-stationary oscillatory processes in systems with both single and multiple degrees of freedom. Mitropolsky' s 1955 monograph, *Non-stationary Processes in Nonlinear Oscillatory Systems*, received well-deserved recognition both in the Soviet Union and abroad, having been translated into English, Japanese, and Chinese. In recent years, the methods presented in that monograph have been further developed and generalized by Mitropolsky and other authors. These results are now reflected in his new monograph, *Problems of the Asymptotic Theory of Non-stationary Vibrations*.

The monograph, totaling 424 pages, consists of eight chapters and an appendix. The first chapter provides examples of typical differential equations with variable coefficients that describe non-stationary processes in nonlinear oscillatory systems with one or more degrees of freedom. It also presents a series of specific examples encountered in various fields of physics and engineering, including electrical and radio engineering (modulation problems), structural engineering (vibrations of bridges and cranes under moving loads and pulsating forces), gyroscopic systems, rocket trajectory calculations during the boost phase, and resonance phenomena during particle acceleration in a synchrophasotron.

The second chapter details the method for constructing asymptotic solutions for a nonlinear differential equation with slowly varying coefficients describing oscillations in a system with one degree of freedom:

$$\frac{d}{dt}\left[m(\tau)\frac{du}{dt}\right]+c(\tau)u=\varepsilon f\left(\tau,u,\frac{du}{dt},\theta\right)$$

where ϵ is a small positive parameter, $\tau=\epsilon t$ is the "slow time," τ represents slowly varying parameters characterizing the oscillatory system (such as mass or stiffness), and $\nu(\tau)$ is the instantaneous frequency of the external perturbing force. The approximate solution is sought as an asymptotic series:

$$u=a\cos\left(\frac{p}{q}\theta+\psi\right)+\varepsilon u_1\left(\tau,a,\theta,\frac{p}{q}\theta+\psi\right)+\varepsilon^2 u_2\ldots$$

where $u_i\left(\tau,a,\theta,\frac{p}{q}\theta+\psi\right)$ are periodic functions of the angles θ and $\frac{p}{q}\theta+\psi$ with period 2π. Here, p and q are coprime integers chosen based on the specific resonance under investigation, while a and ψ are functions of time determined by the following system of differential equations:

$$\frac{da}{dt}=\varepsilon A_1(\tau,a,\psi)+\varepsilon^2 A_2(\tau,a,\psi)+\ldots$$

$$\frac{d\psi}{dt}=\omega(\tau)-\frac{p}{q}\nu(\tau)+\varepsilon B_1(\tau,a,\psi)+\varepsilon^2 B_2(\tau,a,\psi)+\ldots$$

In these equations, $\omega(\tau)=\sqrt{\frac{c(\tau)}{m(\tau)}}$ is the "natural" frequency of the system, $\frac{d\theta}{dt}=\nu(\tau)$ is the instantaneous frequency of the external periodic perturbation, $\tau=\varepsilon t$ is the slow time, and $\omega(\tau)-\frac{p}{q}\nu(\tau)$ characterizes the phase difference between the natural oscillation and the external perturbation, which may vary during the process.

The primary advantage of this method is that the integration of the original equation is reduced to the integration of a system that is significantly simpler and, in many cases, can be solved analytically. Furthermore, this system can be analyzed using phase plane methods. Numerical integration of this system is far simpler and more reliable than direct numerical integration of the original equation. This chapter also outlines convenient techniques for deriving the first and second approximation equations directly from expressions of the work performed by perturbing forces, utilizing the methods of harmonic balance, averaging, and equivalent linearization.

The third chapter provides a detailed exposition of the method for investigating non-stationary oscillations in nonlinear systems with one degree of freedom under external "periodic" perturbing forces with variable frequencies, described by equations of the form:

$$\frac{d}{dt}\left[m(\tau)\frac{dx}{dt}\right] + c(\tau)x = \varepsilon F\left(\tau, \theta, x, \frac{dx}{dt}\right),$$

where $\tau = \varepsilon t$. Typical nonlinear systems are considered, such as the passage through resonance in a nonlinear vibrator under a sinusoidal force with variable frequency and non-stationary regimes involving fractional resonance. The results obtained in this chapter allow for the discovery and detailed study of new phenomena in non-stationary processes that were previously unknown or only observed experimentally.

The fourth chapter presents the single-frequency method for investigating oscillatory systems with many degrees of freedom, described by the equations:

$$\frac{d}{dt}\left\{\sum_{s=1}^{N} a_{rs}(\tau)\dot{q}_s\right\}+\sum_{s=1}^{N} c_{rs}(\tau)q_s = \varepsilon Q_r(\tau, \theta, q_1, \dots, q_N, \dot{q}_1, \dots, \dot{q}_N) \quad (r = 1, 2, \dots, N).$$

Solutions to these equations are sought in the form of series:

$$q_s = \Phi_s^{(1)}(\tau)a\cos(p\varphi + \vartheta) + \varepsilon u_s^{(1)}(\tau, a, \theta, p\varphi + \vartheta) + \varepsilon^2 \dots$$

where $u_s^{(1)}$, $u_s^{(2)}$, $\cdots$are periodic functions of θ and $p\varphi + \vartheta$ with period 2π, and a and ϑ are determined by a system of differential equations:

$$\frac{da}{dt} = \varepsilon A_1(\tau, a, \vartheta) + \varepsilon^2 A_2(\tau, a, \vartheta) + \dots$$

$$\frac{d\vartheta}{dt} = \omega_1(\tau) - \frac{p}{q}\nu(\tau) + \varepsilon B_1(\tau, a, \vartheta) + \varepsilon^2 B_2(\tau, a, \vartheta) + \dots$$

where $\omega_1(\tau)$ is typically the smallest root of the characteristic equation $\det|| - a_{rs}(\tau)\omega^2 + c_{rs}(\tau)|| = 0$, and $\Phi_s^{(k)}(\tau)$ represents the non-trivial solutions of the system of algebraic equations:

$$\sum_{s=1}^{N}[-a_{rs}(\tau)\omega_k^2(\tau) + c_{rs}(\tau)]\Phi_s^{(k)}(\tau) = 0$$

The justification of the single-frequency method is closely linked to the theory of integral manifolds. The particular solutions found, which depend on two arbitrary constants, effectively fill a two-dimensional integral manifold for the system. The method of integral manifolds, originally proposed by N. N. Bogolyubov, has been further developed in the works of Yu. A. Mitropolsky and is now widely applied to solve various problems in the theory of differential equations containing a small parameter. By considering integral manifolds, one can prove theorems that would otherwise require much more stringent conditions on the right-hand sides of the equations. The existence and stability of these manifolds are crucial; if a stable integral manifold exists, the analysis can be concentrated on the solutions lying on this hypersurface rather than the entire phase space.

The appendix provides methods for the numerical integration of nonlinear oscillatory systems using modern computing technology. One of the monograph' s primary achievements is the development of a powerful algorithm for constructing asymptotic approximate solutions for nonlinear differential equations with variable coefficients. This reliable method has revealed new phenomena in non-stationary regimes, particularly regarding the passage through resonance. Previously, such calculations were typically limited to linear systems with a single degree of freedom. Mitropolsky' s work extends this to complex nonlinear systems, analyzing phenomena such as amplitude lag, jumps, and breakdowns. Examples include resonance analysis in crankshafts and gyroscopic devices, as well as the interaction between ordinary and parametric resonance.

In conclusion, Yu. A. Mitropolsky' s monograph, *Problems of the Asymptotic Theory of Non-stationary Vibrations*, represents a fundamental contribution to the theory of nonlinear oscillations and nonlinear differential equations, providing essential tools for both theoretical research and practical engineering applications.

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Note: Figure translations are in progress. See original paper for figures.