Corresponding Member of the USSR Academy of Sciences I. A. KIBEL

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Abstract

Full Text

GEOPHYSICS

Corresponding Member of the USSR Academy of Sciences I. A. KIBEL

ON THE HYDRODYNAMICS OF CLOUD SYSTEMS

One of the urgent problems of dynamic meteorology is the creation of a detailed hydrodynamic theory of individual clouds and cloud systems. In constructing such a theory it must be taken into account that inside a cloud, regardless of whether it is moving or not, intense air motion always occurs; this motion may be treated as pseudoadiabatic, whereas the motion outside the cloud may, in our problem, be regarded as adiabatic. The boundary of the cloud is not known in advance and must be determined simultaneously with the solution of the problem.

For the present we shall restrict ourselves to consideration of the case of a "stationary"cloud, and we shall consider the stage of a developed cloud, when the process may be regarded as approximately steady. The motion accompanying the cloud will be assumed axisymmetric. We take the equations of motion in the form (cylindrical system)

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{\partial \Phi}{\partial r}, \tag{1}$$

$$u\frac{\partial v}{\partial r} + w\frac{\partial v}{\partial z} + \frac{uv}{r} = 0; \tag{2}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial \Phi}{\partial z} + \frac{g}{T_m}T', \tag{3}$$

$$\frac{\partial r\rho_\infty u}{\partial r} + \frac{\partial r\rho_\infty w}{\partial z} = 0; \tag{4}$$

$$u\frac{\partial T'}{\partial r} + w\frac{\partial T'}{\partial z} + (\varepsilon\gamma_a - \gamma)w = 0. \tag{5}$$

Here z is the height above the Earth' s surface; r is the distance from the axis of symmetry; u, v, w are the velocity components along the r axis, around the circle, and vertically, respectively; T' is the deviation of the temperature T from its value $T_\infty(z)$ at a large distance from the cloud; g is the acceleration due to gravity; T_m is the mean temperature of the air column; Φ is the deviation of the geopotential from its standard value; γ_a is the adiabatic gradient; $\gamma = -dT_\infty/dz = \text{const}$; $\varepsilon = 1$ outside the cloud;

$$\varepsilon = \varepsilon_\infty = \left[1 - \frac{0.623}{c_p} L \frac{k}{k-1} \frac{l_{\max}(T_\infty)}{p_\infty T_\infty}\right] \left[1 + \frac{0.623}{c_p} \frac{L}{p_\infty} \frac{dl_{\max}}{dT_\infty}\right]^{-1},$$

where

$$l_{\max}(T_\infty) = 6.1 \cdot 10^{-3} P \exp\left(17.13\, \frac{T_\infty - 273}{T_\infty - 38}\right)$$

(see (1)), where c_p is the heat capacity of air at constant pressure; k is the ratio of heat capacities; L is the latent heat of condensation*;

$p_\infty(z)$ is the standard pressure; P is the pressure at sea level; $\rho_\infty(z)$ is the standard density.

The system (1)–(5) has 4 integrals, which can be constructed after the stream function ψ has been introduced, by (4), from the equalities

$$r \frac{\rho_\infty(z)}{\rho_\infty(0)}\, u = -\frac{\partial \psi}{\partial z}; \qquad r \frac{\rho_\infty(z)}{\rho_\infty(0)} = \frac{\partial \psi}{\partial r}. \tag{6}$$

Then, from (5), we may write

$$T' = -(\gamma_a - \gamma)\,[z - f_1(\psi)] \qquad \text{outside the cloud,} \tag{7}$$

$$T' = -(\gamma_a - \gamma) \left[\int_0^z \frac{\varepsilon_\infty \gamma_a - \gamma}{\gamma_a - \gamma}\, dz - \tilde{f}_1(\psi)\right] \qquad \text{in the cloud,} \tag{8}$$

where f_1 and $\tilde{f}_1$ are arbitrary functions of ψ; finally, from (1)–(3), by eliminating Φ, we obtain outside the cloud

$$\frac{1}{\bar\rho} \frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial \psi}{\partial r}\right) + \frac{1}{r^2 \bar\rho} \frac{\partial}{\partial z} \left(\frac{1}{\bar\rho} \frac{\partial \psi}{\partial z}\right) = -\frac{f_2}{r^2} \frac{df_2}{d\psi} + \frac{g(\gamma_a - \gamma)}{T_m} \left[f_3(\psi) + z \frac{df_1}{d\psi}\right] \tag{9}$$

($\bar\rho = \rho_\infty(z) : \rho_\infty(0)$), where f_3 is a new arbitrary function of ψ, and an analogous equation for the cloud with f_3 replaced by $\tilde{f}_3$ (a new function). We shall not write out the fourth integral—the Bernoulli law.

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* It is assumed that only the liquid phase is present. The generalization to the case in which the solid phase i

In what follows we shall assume that the boundary of the cloud is determined by one of two conditions: either it is a surface that is intersected by the streamlines and that separates the saturated region from the region where the vapor does not saturate the space (the "lower" boundary), or it is a stream surface on which $\psi = \text{const}$ (the "upper" boundary). On the lower boundary we write the saturation conditions, which we represent in the form (see (1))

$$z_{\text{cr}} = m f_1(\psi_{\text{cr}}) + s, \tag{10}$$

where

$$m = [\Gamma + (\gamma_a - \gamma) b] \, (b\gamma_a - g/RT_m)^{-1}; \tag{11}$$

$$s = (bt_0 + \ln(0.38P/q_0 p_0)) \, (b\gamma_a - g/RT_m)^{-1}, \tag{12}$$

with $b = 20.2 : T_m$; $t_0 = T_0 - 273$; T_0 the temperature at $z = 0$ far from the cloud; in this case we assume that the specific humidity q_∞ at a large distance from the cloud is given by the function $q_\infty = q_0 \exp(-\Gamma z)$, where q_0 and Γ are constants.

In addition to (10), we require on the lower boundary: a) continuity of the transition of temperatures; b) continuity of the transition of the vortex $\partial w/\partial r - \partial u/\partial z$; c) continuity of the transition of ψ; d) continuity of the transition of $\partial\psi/\partial z$. Condition a), by (7), (8), and (11), makes it possible to relate $\tilde{f}_1$ and f_1:

$$\tilde{f}_1(\psi) = (1 - m) f_1 + \int_0^{mf_1 + s} \frac{\varepsilon_\infty \gamma_a - \gamma}{\gamma_a - \gamma} \, dz - s. \tag{13}$$

Condition b) relates $\tilde{f}_3$ and f_3 (by (9) and (13)):

$$\tilde{f}_3(\psi) = f_3(\psi) + (mf_1 + s) mn \, df_1/d\psi, \tag{14}$$

where

$$n = 1 - (\varepsilon_\infty \gamma_a - \gamma)/(\gamma_a - \gamma) = (\gamma_a - \varepsilon_\infty \gamma_a)/(\gamma_a - \gamma) \tag{15}$$

(practically, n may be regarded as constant).

We now rewrite (9) in final form, at the same time introducing the dimensionless quantities $\psi, \bar{z}, \bar{r}, \bar{u}, \bar{v}, \bar{w}, f, S, F, \Phi$ from the equalities

$$\psi_0 \bar{\psi} = \psi, \qquad H\bar{z} = z, \qquad H\bar{r} = r, \qquad fH = f_1, \qquad UHF = f_2,$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$H^2\Phi = f_3\psi_0, \qquad HS = s, \qquad U\bar{u} = u, \qquad U\bar{v} = v, \qquad U\bar{w} = w,$$

$$\psi_0 = H^3\sqrt{g(\gamma_a - \gamma)/T_\infty}, \qquad U = \psi_0/H^2 \, *; \qquad H = RT_m/g.$$

Here we assume that $\bar{\rho} = \exp(-\bar{z})$, and also introduce $\zeta = 1 - \bar{e}^{z}$. We obtain (dropping the bars over letters)

$$\frac{1}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\zeta^2} = \Phi + z\frac{df}{d\psi} - \frac{1}{r^2}F\frac{dF}{d\psi}; \tag{16}$$

Fig. 1

$$\frac{1}{\rho}\frac{1}{r}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\zeta^2} = \Phi + \frac{df}{d\psi}\left[m^2nf + mnS + (1-mn)z\right] - \frac{1}{r^2}F\frac{dF}{d\psi}. \tag{17}$$

It remains to choose f, F, and Φ.

We shall restrict ourselves to consideration of two models: a cumulus-cloud model and a model of the central part of a typhoon.

For a cumulus cloud, $F = 0$. We choose $\Phi = -f\,df/d\psi$, $f = \frac{1}{2} - \frac{1}{2}\sqrt{1+4\psi}$ below the cloud and $f = -\frac{1}{2} + \frac{1}{2}\sqrt{1+4\psi}$ above the cloud. In Fig. 1 the boundary of a stationary cloud is given schematically, corresponding to the solution of (16) and (17) for the parameter values $m = 4$, $n = 2.25$, $S^2 = 0.5$. In the calculation we take $\rho = 1$; in doing so we bounded the atmosphere above by a wall of height h ("inversion"); h replaces the quantity RT_m/g. To the right of point A there are descending currents, to the left—ascending ones. The shape of the cloud recalls the appearance of elongated fair-weather clouds.

Fig. 2

For a typhoon (Fig. 2) we took, for the cloud and above the cloud, $F = \omega\psi/(a^2 + \psi^2)$, $\Phi = -f\,df/d\psi$; $f = \frac{1}{2} - \frac{1}{2}\sqrt{1+4\psi}$ below the cloud, $f = -\frac{1}{2} + \frac{1}{2}\sqrt{1+4\psi}$ above the cloud. In Fig. 2 the boundary of cloudiness is given schematically (an approximate solution of (16) and (17)) for $m = 0.8$, $n = 1.25$, $S = 0.15$, $a = 1/16$, $\omega = 3.94$. Again $\rho = 1$ was assumed, and h ("tropopause") was taken instead of H. The form of the "eye of the storm," the "anvil," and certain other details are in general agreement with observations.

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* If $\gamma_a - \gamma < 0$ one must introduce $\psi_0 = \sqrt{g|\gamma_a - \gamma|/T_m}\, H^3$.

Note: Figure translations are in progress. See original paper for figures.

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