A METHOD FOR ANALYZING FINITE AUTOMATA

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Abstract

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CYBERNETICS AND CONTROL THEORY

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A METHOD FOR ANALYZING FINITE AUTOMATA

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This article describes a method for the analysis of finite automata, analogous to the third method for analyzing contact circuits from ([2]).

Let $X = \{x_1, x_2, \dots, x_n\}$ be the (input) alphabet; let V be the set of all (finite) words in this alphabet, including the empty word e; let Δ be the algebra of events in this alphabet, i.e., the set of all subsets of the set V, on which three operations are defined: disjunction $a \cup b$, product ab, and iteration $\langle a \rangle$ ([3]). We shall consider matrices over the algebra Δ. The three operations listed above are naturally transferred to such matrices: disjunction $A \cup B = C$ (where $c_{\alpha\beta} = a_{\alpha\beta} \cup b_{\alpha\beta}$), product $AB = C$ (where $c_{\alpha\beta} = \bigcup_k a_{\alpha k} b_{k\beta}$), and iteration

$$\langle A \rangle = \bigcup_{k=0}^{\infty} A^k$$

(where A^0 is the identity matrix, i.e., the matrix having the event e on the main diagonal and the empty events Λ elsewhere, $A^2 = AA$, $A^3 = AAA, \dots$) ([3]). We shall also denote the iteration $\langle A \rangle$ by $\chi(A)$, and its elements by $\chi_{\alpha\beta}(A)$. By analogy with contact circuits ([2]), we shall interpret a square matrix A as a multipole with poles $z_1, z_2, \dots, z_s$ (s is the order of the matrix A); its element $a_{\alpha\beta}$ ($\alpha, \beta = 1, 2, \dots, s$) will be called the **direct conductance** from pole z_α to pole z_β, and the element $\chi_{\alpha\beta}(A)$ of the iteration the **complete conductance** from z_α to z_β.

Let us note that if the matrix A satisfies the conditions:
1) the events $a_{\alpha\beta}$ contain only one-letter words (not excluding the case $a_{\alpha\beta} = \Lambda$);
2) $a_{\alpha\beta} \cap a_{\alpha\gamma} = \Lambda$ for $\beta \neq \gamma$;
3) $\bigcup_{\beta=1}^{s} a_{\alpha\beta} = X$,

then the matrix A is the transition matrix for a finite automaton with states $z_1, \dots, z_s$.* In this case the complete conductance $\chi_{\alpha\beta}(A)$ is the event that is represented in the automaton by the state z_β, if the state z_α is taken as the initial state of the automaton ([3]).

Let A be an arbitrary square matrix of order s over the algebra Δ. Consider a matrix B of order $(s-1)$, whose elements are determined by the equalities

$$b_{\alpha\beta} = a_{\alpha\beta} \cup a_{\alpha s}\langle a_{ss}\rangle a_{s\beta}, \quad \alpha,\ \beta = 1, 2, \dots, s-1, \tag{1}$$

i.e.

$$B = \begin{bmatrix} a_{1,1} \cdots a_{1,s-1} \\ \cdots\cdots\cdots \\ a_{s-1,1} \cdots a_{s-1,s-1} \end{bmatrix} \cup \begin{bmatrix} a_{1,s} \\ \cdots \\ a_{s-1,s} \end{bmatrix} \langle a_{ss}\rangle \left[a_{s,1} \cdots a_{s,s-1}\right],$$

* If condition 3) is not satisfied, then the automaton will be partial; if condition 2) is not satisfied, then the automaton will be nondeterministic.

which is analogous to formula (1). We shall say that the $(s-1)$-terminal network B is obtained from the s-terminal network A by eliminating the terminal z_s. The following theorem, entirely analogous to the theorem from (2), holds.

Theorem. *When a terminal is eliminated from a multi-terminal network, the complete conductances corresponding to pairs of the remaining terminals remain unchanged, i.e.*

$$\chi_{\alpha\beta}(B) = \chi_{\alpha\beta}(A), \qquad \alpha, \beta = 1, 2, \dots, s-1. \tag{2}$$

Successively eliminating, in some order, the terminals $z_2, z_3, \dots, z_s$, we arrive at a first-order matrix $B = [b_{11}]$. According to the theorem, we shall have $\chi_{11}(A) = \chi_{11}(B) = \langle b_{11}\rangle$. The quantities $\chi_{\alpha\alpha}(A)$, $\alpha = 2, 3, \dots, s$, are found analogously. To find the complete conductance $\chi_{\alpha\beta}$, where $\alpha \neq \beta$, we successively eliminate all terminals except z_α and z_β. As a result we obtain a second-order matrix

$$B = \begin{bmatrix} b_{\alpha\alpha} & b_{\alpha\beta} \\ b_{\beta\alpha} & b_{\beta\beta} \end{bmatrix}.$$

After this the desired conductance can be found by any of the following three formulas:

$$\chi_{\alpha\beta}(A) = \langle b_{\alpha\alpha}\rangle b_{\alpha\beta}\chi_{\beta\beta}(A) = \chi_{\alpha\alpha}(A) b_{\alpha\beta}\langle b_{\beta\beta}\rangle = \chi_{\alpha\alpha}(A) b_{\alpha\beta}\chi_{\beta\beta}(A). \tag{3}$$

Applied to a finite automaton with transition matrix A, the foregoing gives a method for analyzing the automaton.

Suppose we have a finite automaton $z(t) = \varphi(z(t-1), x(t))$ with initial state $z(0) = z_1$, and with output either in the form a) $y(t) = g(z(t))$ (Moore automaton) or in the form b) $y(t) = f(z(t-1), x(t))$ (Mealy automaton). Then the event $\chi(z_1, y_j)$, represented in the automaton by the output signal y_j, is found by the formulas: a) $\chi(z_1, y_j) = \bigcup_k \chi_{1k}(A)$ for a Moore automaton, where k runs through those values for which $g(z_k) = y_j$;* b) $\chi(z_1, y_j) = \bigcup_{(k,i)} \chi_{1k}(A) x_i$ for a Mealy automaton, where (k, i) runs through those pairs of values k and i for which $f(z_k, x_i) = y_j$. However, for analyzing an automaton with output it is convenient to use the following computation scheme.

To the terminals $z_1, z_2, \dots, z_s$, corresponding to the states of the automaton, we add (output) terminals $y_1, y_2, \dots, y_m$, corresponding to the various output signals, and we add additional connections: we connect the terminal z_k with the terminal y_j ($k = 1, \dots, s$; $j = 1, \dots, m$): a) by an edge with conductance e, if $g(z_k) = y_j$ (for a Moore automaton); b) by an edge with conductance x_i, if $f(z_k, x_i) = y_j$ (for a Mealy automaton).

As a result we obtain an $(s + m)$-terminal network C. The event $\chi(z_1, y_j)$, represented in the automaton by the output signal y_j, will be equal to the complete conductance of the multi-terminal network C from the terminal z_1 to the output terminal y_j, i.e. $\chi(z_1, y_j) = \chi_{1,s+j}(C)$. In the actual computation, the rows of the matrix C corresponding to the output terminals need not be written down (these rows will remain zero throughout the entire computation), so that instead of the square matrix C one may operate with a rectangular matrix D of size $s \times (s + m)$. After eliminating the terminals $z_2, z_3, \dots, z_s$, we arrive at the row matrix $[b_{z_1,z_1}, b_{z_1,y_1}, \dots, b_{z_1,y_m}]$, and this completes the analysis process, since

$$\chi(z_1, y_j) = \langle b_{z_1,z_1} \rangle b_{z_1,y_j}, \qquad j = 1, 2, \dots, m. \tag{4}$$

Here, for greater clarity, as indices we have used the designations of the corresponding terminals.

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* If $g(z_1) = y_j$, then $e \in \chi(z_1, y_j)$.

Example (see (3), p. 93). A Mealy automaton with two states 1, 2, two input signals x, y, and two output signals z and v is given by the transition table (Table 1a) and the output table (Table 1b). Find regular expressions for the events $\chi(1, z)$ and $\chi(1, v)$, represented in the automaton by the output signals z and v, with initial state 1.

Table 1

a)

	1	2
x	2	2
y	1	1

b)

	1	2
x	z	v
y	z	v

The transition matrix has the form

$$A = \begin{bmatrix} y & x \\ y & x \end{bmatrix}.$$

Completing it with output columns, we shall have

$$D = \begin{bmatrix} y & x & x \cup y & \Lambda \\ y & x & \Lambda & x \cup y \end{bmatrix}.$$

After eliminating, by formulas (1), state 2, to which the second row and the second column of the matrix D correspond, we arrive at the row matrix

$$[y \cup x\langle x\rangle y, \quad x \cup y \cup x\langle x\rangle\Lambda, \quad \Lambda \cup x\langle x\rangle(x \cup y)],$$

whence, by formulas (4),

$$\chi(1, z) = \langle y \cup x\langle x\rangle y\rangle(x \cup y), \quad \chi(1, v) = \langle y \cup x\langle x\rangle y\rangle x\langle x\rangle(x \cup y).$$

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3. V. M. Glushkov, *Synthesis of Digital Automata*, Moscow, 1962.

Note: Figure translations are in progress. See original paper for figures.