CYBERNETICS AND CONTROL THEORY

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Abstract

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ON THE INVARIANCE OF MODEL-BASED NON-SEARCH SELF-ADJUSTING SYSTEMS

Model-based non-search self-adjusting systems (^1) ensure stability and high quality of control processes when the parameters of the plant vary at a rate comparable with the "rate" of transient processes in the system. We shall show that in these systems, in certain regimes, the error between the output coordinates of the system and of the model is invariant with respect to the control action.

Consider a system described by the equations:

$$\sum_{\alpha=0}^{k} a_\alpha^*(t)\varphi^{(\alpha)} = -b^*(t)\mu \quad \text{—plant;}$$

$$\sum_{\gamma=0}^{r} c_\gamma \mu^{(\gamma)} = \sigma \quad \text{—controller;}$$

$$\sigma = k_b\left(\sum_{i=0}^{n-1} k_i\varphi^{(i)} - k_g g\right) \quad \text{—control law;}$$

$$k_i = \bar{k}_i + k_{iu}z_i, \quad z_i = \int_0^t \left(\varphi^{(i)} - \varphi_M^{(i)}\right)\Phi\left(\varphi_M^{(i)}\right)\operatorname{sign}\varphi^{(i)}\,dt$$

$$\text{—law of variation of the adjustable coefficients;} \tag{1}$$

$$\Phi\left(\varphi_M^{(i)}\right) = \begin{cases} 1, & \text{if } |\varphi_M^{(i)}| > \Delta_i, \\ 0, & \text{if } |\varphi_M^{(i)}| \leqslant \Delta_i, \end{cases} \quad \text{—nonlinear function;}$$

$$\sum_{\xi=0}^{n} d_\xi \varphi_M^{(\xi)} = g \quad \text{—model,}$$

where $r+k=n$; φ is the controlled coordinate of the plant; μ is the coordinate of the regulating element; σ is the control law; φ_M is the coordinate of the model; $a_\alpha^*(t)$, $b^*(t)$ are variable coefficients of the plant; k_b is the overall coefficient of the controller; c_γ, $\bar{k}_i$, k_{iu}, d_ξ, Δ_i are constant coefficients; g is the control action (an analytic function bounded in modulus, holomorphic at $t=t_0$).

We shall assume that $a_\alpha^*(t)$ and $b^*(t)$ varied in an arbitrary manner and then, beginning from some $t=T$, remain constant. Then the equations of the perturbed motion of the system, upon linearization of the terms $\varphi^{(i)}z_i$ and for $|\varphi_M^{(i)}|>\Delta_i$, will be

$$\operatorname{sign}(\varepsilon+\varphi_M)\sum_{\alpha=0}^{k}a_\alpha^*D^{\alpha+1}\Delta z_0+b^*\mu=-\sum_{\alpha=0}^{k}a_\alpha^*D^\alpha\varphi_M,$$

$$\sum_{\gamma=0}^{r}c_\gamma D^\gamma\mu-\sigma=0,$$

$$k_b\operatorname{sign}(\varepsilon+\varphi_M)\left[\sum_{i=0}^{n-1}(\bar{k}_i+k_{iu}\tilde{z}_i)D^{i+1}+\sum_{i=0}^{n-1}k_{iu}(D^i\varphi_M)\operatorname{sign}D^i(\varepsilon+\varphi_M)D^i\right]\Delta z_0-$$

$$-\sigma=\left[k_{g0}k_bd_{nD}^n+k_b\sum_{i=0}^{n-1}(\bar{k}_i+k_{iu}\tilde{z}_i-k_{g0}d_i)D^i\right]\varphi_m, \qquad (2)$$

where $\varepsilon=\varphi-\varphi_m'$, $z_i=\tilde{z}_i+\Delta z_i$, $\tilde{z}_i=\text{const}$, $i=0,1,\dots,n-1$,

$$\Delta z_i=D^i\Delta z_0\operatorname{sign}D^i(\varepsilon+\varphi_m)\operatorname{sign}(\varepsilon+\varphi_m),\quad i=1,2,\dots,n-1.$$

System (2) is nonlinear with variable coefficients. Suppose that the control action g is such that, for

$$|z_i(t)-\tilde{z}_i|<\bar{z}_i,\quad t>\bar{T},\quad \bar{T}>T \qquad (3)$$

the inequalities are satisfied

$$|\varphi_m-\varphi_{m_0}|<\delta,\quad |\varphi_m^{(i)}|<\delta,\quad i=1,2,\dots,n-1, \qquad (4)$$

where $\bar{z}_i,\delta$ are small positive quantities, $\varphi_{m_0}=\text{const}\neq0$, $|\varepsilon|<\varphi_m$.

Then, neglecting in the coefficients of $D^i\Delta z_0$, $i=1,2,\dots,n-1$, in the last equation (2) the terms containing the factor δ, we obtain the linear system (since φ_{m_0} does not change sign) with constant coefficients ($t>\bar{T}$)

$$\operatorname{sign}\varphi_{m_0}\sum_{\alpha=0}^{k}a_\alpha^* D^{\alpha+1}\Delta z_0+b^*\mu=-\sum_{\alpha=0}^{k}a_\alpha^* D^\alpha\varphi_m,$$

$$\sum_{\gamma=0}^{r}c_\gamma D^\gamma\mu-\sigma=0,\quad r+k=n,$$

$$k_b\operatorname{sign}\varphi_{m_0}\left[\sum_{i=0}^{n-1}(\bar{k}_i+k_{iu}\tilde{z}_i)D^{i+1}+k_{0u}\varphi_{m_0}\operatorname{sign}\varphi_{m_0}\right]\Delta z_0-\sigma=$$

$$=\left[k_{g0}k_b d_{nD}^n-k_b\sum_{i=0}^{n-1}(\bar{k}_i+k_{iu}\tilde{z}_i-k_{g0}d_i)D^i\right]\varphi_m. \tag{5}$$

Represent (5) in the form of a matrix equation

$$Ax=F, \tag{6}$$

where $A=\|a_{ij}\|$, $i,j=1,2,3$, is a polynomial or D-matrix of order 3; x,F are column matrices, with

$$x_1=\Delta z_0,\quad x_2=\mu,\quad x_3=\sigma,$$

$$F_1=-\sum_{\alpha=0}^{k}a_\alpha^* D^\alpha\varphi_m,\quad F_2=0,$$

$$F_3=\left[k_{g0}k_b d_{nD}^n-k_b\sum_{i=0}^{n-1}(\bar{k}_i+k_{iu}\tilde{z}_i-k_{g0}d_i)D^i\right]\varphi_m,$$

$$a_{11}=\operatorname{sign}\varphi_{m_0}\sum_{\alpha=0}^{k}a_\alpha^* D^{\alpha+1},\quad a_{12}=b^*,\quad a_{13}=0,$$

$$a_{21}=0,\quad a_{22}=\sum_{\gamma=0}^{r}c_\gamma D^\gamma,\quad a_{23}=-1,$$

$$a_{31}=k_b\operatorname{sign}\varphi_{m_0}\left[\sum_{i=0}^{n-1}(\bar{k}_i+k_{iu}\tilde{z}_i)D^{i+1}+k_{0u}\varphi_{m_0}\operatorname{sign}\varphi_{m_0}\right],$$

$$a_{32}=0,\quad a_{33}=-1.$$

We reduce (6) to Hermite' s canonical form ([2]). As the canonicalizing matrix λ we choose

$$\lambda = \|\lambda_{ij}\| = \left\| \begin{matrix} 0 & 0 & 1 \\ \dfrac{1}{b^*} & 0 & 0 \\ \dfrac{1}{c_r a_k^*} \sum_{\gamma=0}^{r} c_\gamma D^\gamma & -\dfrac{b^*}{c_r a_k^*} & \dfrac{b^*}{c_r a_k^*} \end{matrix} \right\|. \tag{7}$$

Multiplying the augmented matrix of system (6) by λ on the left, we find the augmented matrix of Hermite' s canonical form and the corresponding system

$$\begin{aligned} \lambda_{31}a_{31}x_1 + 0x_2 + \lambda_{13}a_{33}x_3 = \lambda_{13}F_3, \\ \lambda_{21}a_{11}x_1 + \lambda_{21}a_{12}x_2 + 0x_3 = \lambda_{21}F_1, \\ (\lambda_{31}a_{11} + \lambda_{33}a_{31})x_1 + 0x_2 + 0x_3 = \lambda_{31}F_1 + \lambda_{33}F_3. \end{aligned} \tag{8}$$

A necessary and sufficient condition for absolute invariance of x_1, and consequently also of $\varepsilon = \varphi - \varphi_\mathrm{m}$, with respect to g or φ_m, is the condition

$$\lambda_{31}F_1 + \lambda_{33}F_3 = 0. \tag{9}$$

Expanding (9), we obtain

$$d_n = \frac{1}{\varkappa k_{g0}}, \qquad \tilde{z}_i = \frac{\varkappa k_{g0} - \varkappa k_i - a_i}{\varkappa k_{iu}}, \tag{10}$$

where $\varkappa = k_b b^*/c_r a_k^*$; a_i are the coefficients of the polynomial

$$\sum_{i=0}^{n} a_{iD}^{i} = \frac{1}{c_r a_k^*} \sum_{\alpha=0}^{k} a_\alpha^* D^\alpha \cdot \sum_{\gamma=0}^{r} c_\gamma D^\gamma, \qquad a_n = 1. \tag{11}$$

It is easy to show that, for $\varkappa = \mathrm{const}$, in the steady state in the regime under consideration $z_{i\,\mathrm{st}} = \tilde{z}_i$. Hence the error $\varepsilon = \varphi - \varphi_\mathrm{m}$ in a searchless self-adjusting system with a model is absolutely invariant with respect to the control action $g(t)$ in the regime when $a_\alpha^*(t)$, $b^*(t)$, after changing in an arbitrary manner, then remain constant. The condition $\varkappa = \mathrm{const}$ is achieved by means of a special self-adjustment loop for the overall gain coefficient of the regulator k_b, which can be constructed on the basis of monitoring the amplitude-frequency characteristic of the closed system at one point.

Since z_i attains the values $z_{i\,\mathrm{st}}$ as $t \to \infty$, absolute invariance can be attained only at $t = \infty$. However, this difficulty is easily avoided if it is assumed that the transient processes end when $|\varepsilon| < \bar{\varepsilon}$ and $|z_i - z_{i\,\mathrm{st}}| < \bar{z}_i$, where $\bar{\varepsilon}, \bar{z}_i$ are small quantities. The indicated formulation of the invariance problem is, in essence, "invariance up to ε."

The necessary realizability condition ([3]) for the relations (10) is satisfied, and the system will be coarse for bounded $|\varphi_{\mathrm{m}}|$.

In conclusion, we note that the invariance conditions (10) retain their form also for an arbitrary control action $g(t)$.

Thus, in studying and selecting the structure of searchless self-adjusting systems with a model, a new approach can be applied, connected with the problem of invariance in the theory of automatic control.

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Note: Figure translations are in progress. See original paper for figures.