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Abstract

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EXPERIMENTAL VERIFICATION OF THE APPLICABILITY OF PASCAL' S LAW IN NONUNIFORMLY HEATED GASES

As is well known, the pressure in a gas that is in a state of equilibrium is distributed isotropically. This proposition can be rigorously justified thermodynamically. However, the thermodynamic conclusion evidently loses its force when a temperature gradient is present in the gas. A solution of the question, however, can be obtained from the kinetic theory of gases. As was shown in work ([1]), the expression for the tensor of thermal stresses in a gas, given by Maxwell ([2]), indicates a violation of Pascal' s law in a gas through which a heat flux Q flows. Namely, for $Q = \text{const}$ (i.e., in the absence of radiant heat exchange in the volume of the gas), the relation

$$P_{xx} - P_{zz} = 2.1\,\frac{\eta^2}{\rho T^2}\,\frac{Q^2}{\chi^2} = 2.1\,\frac{\eta^2}{\rho T^2}\,(\operatorname{grad} T)^2 \tag{1}$$

holds (where P_{xx} and P_{zz} are the pressures in the direction of the x or z axis on areas perpendicular to the corresponding axes; η, χ, and ρ are the viscosity, thermal conductivity, and density of the gas; T is the absolute temperature, the x axis is parallel to the temperature gradient), expressing the anisotropy of the tensor of thermal stresses. However, a more accurate expression for the tensor of thermal stresses, given in ([3]), leads to the opposite conclusion ([1]):

$$P_{xx} - P_{zz} = 0. \tag{2}$$

In view of the fact that, according to ([4–10]), phenomena in the volume of the gas are the basic factor responsible both for thermophoresis and for the thermomolecular pressure difference (in wide capillaries), it is important to determine by direct measurements which of the two relations, (1) or (2), proves to be correct. For this purpose we constructed the apparatus shown in Figs. 1 and 2. The principal part of the metal apparatus shown in Fig. 1 is the refrigerator 7, which is a hollow copper cylinder whose outer end surface has been optically polished to the cleanliness of surface $ $8. The refrigerator 7 is located inside a brass housing 5, which has, polished at the end, a cylindrical projection A 16

Fig. 1

Figure 1: Fig. 1

mm in diameter and 4.993 ± 0.004 mm high. A porcelain ring 9 is fitted onto the projection; its end surfaces are optically polished. The height of the rings was measured with an IKV optimeter to an accuracy of $0.5\,\mu$, and only rings with a maximum deviation from the mean height of $\pm 3\,\mu$ were used in the work.

The copper refrigerator 7, by means of a brass bellows 4, which plays the role of a spring and at the same time serves to seal the space inside the housing 5, was pressed against the end of the porcelain ring 9 fitted onto projection A. As a result, between the polished surfaces of projection A and the copper refrigerator 7 there is formed a plane-parallel gap SS, whose height h_0 is determined by the formula $h_0 = h_2 - h_1 = (h_2 - 4.993)$ mm, where h_2 is the height of the porcelain ring and h_1 is the height of projection A. Using a set of 10 porcelain rings of different height, we obtained different heights of the gap SS. In addition, in a number of experiments the gap SS was produced not with the aid of the porcelain ring 9, but with the aid of three identical pieces of mica placed on projection A,

to which the refrigerator 7 was pressed. In this case the height of the gap SS was equal to the thickness of the mica, and no additional error arose from inaccurate measurement of the height of the projection A.

The annular rubber gasket 3 provides a vacuum seal for the space under the brass cover 2. If, instead of the bellows 1, we had used a rigid connection between cover 2 and refrigerator 7, then, owing to the unavoidable tilting when the bolts were tightened, the gap SS would not have been plane-parallel.

The temperature difference between the refrigerator 7 and the projection A is produced by means of a mixture of solid carbon dioxide with acetone ($t \cong -76^{\circ}$) located in the refrigerator 7, and by hot water supplied from an ultrathermostat through the fittings 12 and 10 into the body 5. The temperature difference between the refrigerator 7 and the projection A was measured by thermocouples with an accuracy of up to 0.5°.

Fig. 1. Apparatus for creating a temperature field. 1, 11—vacuum fittings, 2—cover, 3—rubber gasket, 4—bellows, 5—body, 6, 8—Teflon bushings, 7—refrigerator, 9—porcelain ring, 10, 12—water fittings, A—projection, C—hole

In order to provide thermal insulation between the body and the refrigerator 7, we used Teflon bushings 8 and 6. The diameter of bushing 6 is 1 mm larger than the diameter of the refrigerator 7, so that in the absence of a temperature gradient the pressure in the gap SS was equal to the pressure in the space under cover 2, connected to the vacuum fitting 1. To connect this space with the gap SS, two grooves of depth 0.5 mm and width 1 mm were cut in the porcelain rings 9. At the center of projection A of the body 5 a hole of diameter

2 mm was drilled, while between the bottom of this hole and the end surface of projection A there remained a layer of brass about 0.1 mm thick. This layer was pierced with a thin steel needle.* The diameter of the resulting through hole C proved to be close to $30\,\mu$. Such a small hole diameter is necessary so that the temperature field in the gap SS is not distorted by the presence of hole C.

In the gap SS the lower plate is hotter than the upper one, which may cause gravitational convection in the gap. However, Rayleigh ([11]) found that convection between parallel horizontal plates, when the lower of them is heated, arises only at a certain critical value of the dimensionless quantity Ra, which was called the Rayleigh parameter:

$$\mathrm{Ra} = \rho^2 c_p h_0^3 g \beta \Delta T / \eta \chi, \tag{3}$$

where χ and c_p are the thermal conductivity and specific heat of the gas (in our case, air); β is the coefficient of volume expansion; h_0 is the distance between the plates. In the monograph ([12]), on the basis of a number of experimental studies, the conclusion is drawn that for $\mathrm{Ra} < 1620$ there is no convection between the plates. In our case, for $\Delta T = 150^\circ$ and $h_0 = 0.3$ mm, $\mathrm{Ra} \cong 0.15 \ll 1620$, i.e., convection in the gap SS did not arise in our experiments, despite the fact that the air in the gap was heated from below.

The gap SS, through the vacuum fittings 1 and 11, is connected to a glass apparatus (Fig. 2) serving to measure the pressure in the system and the pressure difference between fittings 1 and 11. The latter represents the anisotropy of the air pressure in the thermal field, since in fitting 11

* We express our gratitude to N. D. Loginov for his excellent execution of this work.

the pressure is measured at a surface perpendicular to the temperature gradient, and in fitting 1 at a surface parallel to the gradient, since fitting 1 is connected with slit SS through the gap between the Teflon bushing 6 and the refrigerator 7 and through grooves in the porcelain ring 9 (it should be taken into account that along this path there are no appreciable temperature gradients and, consequently, the pressure is transmitted here isotropically in all directions).

Between the metallic apparatus T and the glass measuring apparatus there are refrigerators X_1 and X_2, cooled by a mixture of dry ice and acetone, in which water vapor is frozen out. The glass apparatus consists of manometers P_1 and P_2 and a volume $V = 5$ liters. The manometers are filled with PFMS-2 oil (density $\rho = 1.030$ g/cm^3; saturated-vapor pressure not higher than $7 \cdot 10^{-7}$ mm Hg at 20°), which had previously been kept for 24 hours under vacuum to remove adsorbed air. In order to reduce the leakage of air into the system, glass tubes and very short sections of vacuum hose were used to connect the glass measuring apparatus and the metallic apparatus T.

Fig. 2. Schematic of the setup. P_1 and P_2—manometers, K_1–K_5—valves, T–

metallic apparatus for creating the temperature field, X_1, X_2—refrigerator traps, M—microscope, L—condenser, O—illuminator.

For measuring the pressure, manometer P_1 is used. In the movable part of valve K_3 there is one opening. In the first position of valve K_3, the left and right arms of manometer P_1 are connected to one another, and with the setting of the three-way valve K_4, when all three tubes connected to it are connected to one another (i.e., manometers P_1 and P_2 are switched off), the system is pumped down to the required pressure. The volume V makes it possible to evacuate the system more smoothly. In the second position of valve K_3, the two arms of manometer P_1 are separated, and the right arm of manometer P_1 and simultaneously the left arm of manometer P_2 are connected with fitting 11 of the metallic apparatus. The fore-vacuum pump evacuated air from the left arm of manometer P_1 to a pressure $p \approx 7 \cdot 10^{-2}$ mm Hg, which was measured with an LT-2 thermocouple and a VT-2A thermocouple vacuum gauge. The pressure in the right arm of manometer P_1, i.e., the pressure in the metallic apparatus, was

$$p = (7 \cdot 10^{-2} + 1.03 \cdot h') \text{ mm Hg},$$

where h' is the difference between the oil levels in the arms of manometer P_1. Since the diameter of the right arm was much larger than the diameter of the left, we neglected the lowering of the oil level in the right arm. At the pressures used, $p \gtrsim 7$ mm Hg, the correction $7 \cdot 10^{-2}$ mm Hg was also neglected.

For measuring the anisotropy of the air pressure in the temperature field (i.e., the pressure difference between fittings 1 and 11), differential manometer P_2 is intended. When measuring the pressure difference, valve K_4 separates the left and right arms of manometer P_2 and connects the right arm with fitting 1, while valve K_3 connects fitting 11 with the left arm of manometer P_2. The oil level in the right arm of manometer P_2 was measured with the aid of microscope M (MIR-2 microscope, 3.7× objective) and an MOV-1-15 screw ocular micrometer; the accuracy of measuring the oil level was 2.2 μ.

Gas leakage (due to leaks in the system or the presence of volatile residues in the system), measured by manometer P_1, proved to be no more than $3 \cdot 10^{-2}$ mm Hg/min. The displacement of the oil level in manometer P_2 due to gas leakage was no more than 10^{-2} mm Hg/min. In the case of creating a pressure difference

between the right and left arms of manometer P_2 after the manometer was shut off by tap K_4, the oil returned to the zero level, which differed from the initial one by no more than $10\,\mu$ (i.e., by $\approx 10^{-2}$ mm water column).

The pressure anisotropy was measured at pressures $p = 7 \div 100$ mm water column, with a gap height h_0 of SS from 0.07 to 0.3 mm, at three temperature differences: $\Delta T = 100^\circ, 115^\circ$, and 135° (this corresponds to a range of temperature gradients from 3000 to 19 000 deg/cm). The minimum pressures used

($p \sim 7$ mm water column) were chosen so that the mean free path was smaller than the gap height, since the theoretical calculations carried out by Yalamov and Deryagin ([1]), both on the basis of Maxwell' s thermal-stress tensor ([2]) and on the basis of the Chapman–Enskog tensor ([3]), are applicable only when $\lambda \ll h_0$. Some measurement conditions are given in Table 1. In the column $\Delta p_{\text{theor. Maxw}}$ the value of the pressure anisotropy is given as theoretically obtained by Yalamov and Deryagin ([1]) on the basis of Maxwell' s thermal-stress tensor ([2]) (from the Chapman–Enskog thermal-stress tensor, as is seen from ([1]), there follows the absence of pressure anisotropy in air). The experimentally measured pressure difference (see Table 1), within the limits

Table 1

p, mm water col.	$\Delta p_{\text{exp}} \cdot 10^3$, mm water col.	$\Delta p_{\text{Maxw.}} \cdot 10^3$, mm water col.	p, mm water col.	$\Delta p_{\text{exp}} \cdot 10^3$, mm water col.	$\Delta p_{\text{Maxw.}} \cdot 10^3$, mm water col.
$h_0 = 100\,\mu$, $\Delta T = 115^\circ$, grad T = 11500 deg/cm	$h_0 = 100\,\mu$, $\Delta T = 115^\circ$, grad T = 11500 deg/cm	$h_0 = 100\,\mu$, $\Delta T = 115^\circ$, grad T = 11500 deg/cm	$h_0 = 200\,\mu$, $\Delta T = 135^\circ$, grad T = 6750 deg/cm	$h_0 = 200\,\mu$, $\Delta T = 135^\circ$, grad T = 6750 deg/cm	$h_0 = 200\,\mu$, $\Delta T = 135^\circ$, grad T = 6750 deg/cm
7	7.2	1340	7	10.2	462
10	7.2	936	10	11.8	324
15	−3.3	625	15	10.2	216
20	−8.2	468	20	−7.2	162
40	12.1	234	40	−6.6	81
80	−9.5	117	80	9.5	40
100	−6.2	94	100	−6.2	33

of the experimental error (each measurement was carried out over 2 min–during this time the error due to leakage may amount to $\sim 2 \cdot 10^{-2}$ mm water column), proved in all cases to be equal to zero. Thus, in agreement with the result obtained by Yalamov and Deryagin ([1]) on the basis of the Chapman and Enskog thermal-stress tensor ([3]), Pascal' s law (the pressure in a gas is isotropic in all directions), which can strictly be derived only for a system in thermal equilibrium, proved applicable also in the presence of a temperature field. At the same time, from Maxwell' s thermal-stress tensor ([2]) it follows that, under our conditions, there should exist a pressure anisotropy of air considerably exceeding the possible error of our experiment. The experimentally observed absence of such anisotropy proves the insufficiency of the thermal-stress tensor obtained by Maxwell ([2]), and the necessity of taking into account in it an additional term, as was done in ([3]).

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Note: Figure translations are in progress. See original paper for figures.