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Abstract

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PHYSICS

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ON THE ROLE OF DIFFUSION OF INVERSE POPULATION IN THE GENERATION OF INDUCED RADIATION

(Presented by Academician I. V. Obreimov, 31 X 1964)

In early works ([1]) on the theory of generation of induced radiation in multimode resonators, the spatial inhomogeneity of the radiation field inside the resonator and, consequently, the spatial inhomogeneity of the inverse population in the sample are not taken into account; this implicitly assumes the presence of an infinitely large rate of diffusion of the inverse population. Conversely, in ([2]) it is assumed that the rate of diffusion of the inverse population is so small that it need not be taken into consideration in analyzing the generation process. However, a more rigorous treatment, based on the equation for the inverse population with allowance for the diffusion term, shows that diffusion may prove to be a very substantial factor (especially at low temperatures), affecting both the spectral composition and the number of radiation quanta.

Indeed, if it is assumed that diffusion of the excitation occurs in a linear chain of active centers with distance R between neighboring centers, then for $R \ll \lambda$, where λ is the wavelength of the radiation determining the linear dimensions of the inhomogeneity of the inverse population $n(z,t)$, the term allowing for diffusion will have the form $k\,\partial^2 n/\partial z^2$. The diffusion coefficient k turns out to be $\sim WR^2$, where W is the frequency of exchange of excitation between neighboring centers.

In view of this, the system of equations describing the process of generation of induced radiation in the model of a plane resonator, taking into account only axial modes, may be written in the form:

$$\frac{dn}{dt} = -\frac{n-n_0}{\tau} - \sum_{(i)} Dg_i nP_i + k\frac{\partial^2 n}{\partial z^2},$$

$$\frac{dN_i}{dt} = -\gamma_i N_i + \int_0^L Dg_i nP_i\,dz, \quad (1)$$

where τ is the time of spontaneous decay; L is the length of the resonator; $\gamma_i \approx \gamma$ is the relative number of photon losses in the i-th mode per unit time; n_0 is the stationary inverse population in the absence of induced radiation; g_i is the ordinate of the normalized luminescence line shape at the frequency of the i-th resonator mode; D is a quantity proportional to the Einstein coefficient; $P_i(z,t) = N_i(t)\left(1 - \cos\frac{2\pi m_i}{L} z\right)$ is the energy density of the i-th axial mode at the point z.

In the stationary case, dn/dt and dN_i/dt are equal to zero, and the equation for the inverse population, under the condition $D\tau \sum g_{iN} i \ll 1$, which corresponds to weak pumping, and the boundary condition $n(0) = n(L)$, has the solution

$$n = n_0 \left(1 - D\tau \sum g_{iN} i + D\tau \sum \frac{g_{iN} i}{1 + d_i} \cos \frac{2\pi m_i}{L} z\right), \tag{2}$$

where $d_i = (2\pi m_i/L)^2 k\tau$ (in what follows we assume all $d_i \approx d$).

If $2j+1$ modes enter into generation, then the number of photons in the i-th mode is

$$N_i = \frac{2[1 + (j-i+1)^2 \beta(1+d)]}{aD\tau g} \times$$

$$\times \frac{\alpha - 1 - \beta\{[2(2j+1)(1+d)+1](j-i+1)^2 + 2(1+d)j(j+1)(2j+1)/3\}}{2(2j+1)(1+d)+1}, \tag{3}$$

where $g = (\pi\Delta\nu)^{-1}$, $\alpha = Dgn_0L/\gamma$, $\beta = (\delta\nu/\Delta\nu)^2$, $\delta\nu = c/2L$, and $2\Delta\nu$ is the half-width of the luminescence line. The number of modes generated for given parameters is determined from the condition $N_{2j+2} = 0$. We obtain the equation:

$$\frac{\alpha - 1}{2\beta(1+d)} = (j+1)^2 \left(2j + 1 + \frac{1}{2}\frac{1}{1+d}\right) - \frac{j(j+1)(2j+1)}{3}. \tag{4}$$

In the case when j is large, it follows from equation (4) that

$$j \approx \sqrt[3]{\frac{3}{8}(\alpha - 1)/\beta(1+d)}, \tag{5}$$

i.e., as diffusion increases, the number of generated modes decreases, and consequently the emission spectrum narrows.

In the case when $\alpha \le 1 + \beta(3 + 2d)$, only one mode enters into generation, and

$$N_1 = \frac{1}{D\tau g}\frac{1+d}{d+{}^3/_2}\left(1-\frac{1}{\alpha}\right). \tag{6}$$

An estimate of the diffusion coefficient $k \sim WR^2 = e^4 f_{n0}^2/m^2\nu^2 R^4 \Delta\nu$ (2, 3), where e and m are the charge and mass of the electron, ν is the operating frequency, $2\Delta\nu$ is the half-width of the luminescence line, and f_{n0} is the oscillator strength of the working transition, gives for ruby at room temperature ($\tau \approx 3 \cdot 10^{-3}$ sec) a value of order 10^{-9} cm^2/sec; in this case $d \approx 0.1$, while at liquid-nitrogen temperature, owing to the fact that $\Delta\nu$ decreases by an order of magnitude, $d \approx 1$. As d changes from 0 to 1, with $\alpha \leq 1 + 3\beta$, the number of photons, as follows from relation (6), increases by 20%.

The results obtained above pertain to media with homogeneous broadening of the luminescence line of the active centers. In media with inhomogeneous broadening, if one assumes that the natural width of the luminescence line of a given active center is smaller than the frequency difference between neighboring axial modes, and also allows for the absence of interaction between centers operating at different frequencies (such conditions can probably be realized at sufficiently low temperatures), then generation in different modes occurs independently, and diffusion in each mode will then be described by formula (6), in which α depends on the frequency. It is interesting to note that, since $k \sim R^{-4}$, owing to diffusion one may expect (other conditions being equal) a narrowing of the emission spectrum as the concentration of active centers in the crystal is increased.

In conclusion, we express our gratitude to Academician I. V. Obreimov for his attention to this work.

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Note: Figure translations are in progress. See original paper for figures.