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Abstract

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THEORY OF ELASTICITY

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ON THE CALCULATION OF CYLINDRICAL SHELLS LOADED ALONG LINES

In paper ([1]), asymptotic formulas are given for the forces and moments in the middle surface of a shell, increasing without bound in the vicinity of the points of application of concentrated forces. Analogous formulas were obtained in ([2]) for the case when the shell is loaded by forces uniformly distributed over a small rectangular area and a small segment of a generator or of an arc of circumference. Below we consider an infinitely long thin elastic circular cylindrical shell loaded along a segment of a generator by arbitrary distributed loads. Asymptotic formulas are obtained for the forces in the middle surface of the shell, increasing without bound in the vicinity of the ends of the segment.

Let $x = R\xi$, $y = R\theta$ be, respectively, the longitudinal and circumferential coordinates associated with the middle surface of the shell; R the radius of the middle surface; $q_1(\xi)$, $q_2(\xi)$ the intensities of the external loads distributed over the segment $\theta = 0$, $a \leq \xi \leq b$, of the generator and directed, respectively, along the generator and along the tangent to the circumference.

In determining the forces in the middle surface of the shell due to the loads $q_1(\xi)$ and $q_2(\xi)$, as Green' s functions we shall use the corresponding solutions for concentrated forces, first separating from these solutions the principal values in the form of the asymptotic representations ([1]). Then each desired force can likewise be represented as the sum of a principal value and a certain bounded and continuous function, which we shall subsequently disregard. The principal values, which may undergo a discontinuity upon crossing the loaded segment (a, b) and increase without bound in the vicinity of its ends, are represented by linear combinations of the following integrals:

$$F_{1j}(z) = \int_a^b \frac{q_j(t)}{\rho} \cos\alpha \, dt, \qquad F_{2j}(z) = \int_a^b \frac{q_j(t)}{\rho} \sin\alpha \, dt,$$

$$F_{3j}(z) = \int_a^b \frac{q_j(t)}{\rho} \cos\alpha \cos 2\alpha \, dt, \qquad F_{4j}(z) = \int_a^b \frac{q_j(t)}{\rho} \sin\alpha \cos 2\alpha \, dt \qquad (1)$$

$$(j = 1, 2);$$

$$\rho = |z - t|, \qquad \alpha = \arg(z - t), \qquad z = \xi + i\theta,$$

$$\frac{1}{\rho}\cos\alpha = -\frac{1}{2}\left(\frac{1}{t-z} + \frac{1}{t-\bar{z}}\right), \qquad \frac{1}{\rho}\sin\alpha = \frac{1}{2i}\left(\frac{1}{t-z} - \frac{1}{t-\bar{z}}\right), \qquad (2)$$

$$\cos 2\alpha - 1 = i\theta\left(\frac{1}{t-z} - \frac{1}{t-\bar{z}}\right).$$

The asymptotic representations of the functions (1) in the vicinity of the ends a and b may be obtained by using the properties of Cauchy-type integrals ([5]). Thus, if $q_1(t)$ and $q_2(t)$ are arbitrary continuous functions, bounded at the ends of the segment, then the integrals $F_{2j}(z)$ and $F_{4j}(z)$ are bounded, while the asymptotic-

the asymptotic representations of the integrals $F_{1j}(z)$ and $F_{3j}(z)$ will be

$$F_{1j}(z) \approx F_{3j}(z) \approx \pm q_j(c)\ln\rho_c. \qquad (3)$$

Here and below, ρ_c is the distance from the point c, coinciding either with a or with b, to the point z at which the forces are being sought. In equalities containing two signs on the right-hand side, the upper sign is taken for the neighborhood of the point a, and the lower one for b. The asymptotic formulas for the forces in the shell have the form

$$T_1(z) \approx \mp\frac{3+\nu}{4\pi}q_1(c)\ln\rho_c, \qquad T_2(z) \approx \pm\frac{1-\nu}{4\pi}q_1(c)\ln\rho_c,$$

$$S_1(z) = -S_2(z) \approx \mp\frac{1-\nu}{4\pi}q_2(c)\ln\rho_c. \qquad (4)$$

The longitudinal tensile forces T_1 and the circumferential forces T_2 due to the load $q_2(\xi)$, as well as the shearing forces $S_1 = -S_2$ due to the load $q_1(\xi)$, will be bounded (the notation for the forces is taken from (4)). It follows from formulas (4) that, if the functions $q_1(\xi)$ and $q_2(\xi)$ vanish at the ends of the segment (ab), then all forces in the shell will be bounded. Suppose that the functions $q_1(\xi)$ and $q_2(\xi)$ in the neighborhood of the ends a and b have, respectively, the form ($q_j^0 = \mathrm{const}$),

$$q_j(\xi) = q_j^0/\sqrt{\xi - a}, \qquad q_j(\xi) = q_j^0/\sqrt{b - \xi} \qquad (j = 1, 2), \qquad (5)$$

i.e., they increase without bound. Such a character of singularities is possessed, for example, by the tangential forces $q_1(\xi)$ transmitted from stringers attached to the cylindrical shell and loaded at their ends by longitudinal forces (3). The asymptotic representations of the integrals (1) in the neighborhood of the ends of the segment (a,b) in this case will be

$$F_{1j}(z) \approx -\frac{\pi}{\sqrt{\rho_c}} q_j^0 \sin\frac{\alpha_1}{2},$$

$$F_{3j}(z) \approx -\frac{\pi q_j^0}{\sqrt{\rho_c}} \sin\frac{\alpha_1}{2}\left(1+\cos\frac{\alpha_1}{2}\cos\frac{3\alpha_1}{2}\right), \tag{6}$$

$$F_{2j}(z) \approx \frac{\pi q_j^0}{\sqrt{\rho_c}}\cos\frac{\alpha_1}{2}, \qquad F_{4j}(z) \approx \frac{\pi q_j^0}{2\sqrt{\rho_c}}\sin\alpha_1 \sin\frac{3\alpha_1}{2},$$

where α_1 is the angle between the segment (a,b) and the vector connecting the point a or b with the point z. This angle is measured from the segment (a,b) counterclockwise if z belongs to the neighborhood of the point a, and clockwise if z belongs to the neighborhood of the point b. The forces in the shell near the ends of the segment (a,b) in this case are expressed by the asymptotic formulas

$$T_1(z) \approx \pm\frac{q_1^0}{4\sqrt{\rho_c}}\sin\frac{\alpha_1}{2}\left[(3+\nu)+(1+\nu)\cos\frac{\alpha_1}{2}\cos\frac{3\alpha_1}{2}\right],$$

$$T_2(z) \approx \mp\frac{q_1^0}{4\sqrt{\rho_c}}\sin\frac{\alpha_1}{2}\left[1-\nu+(1+\nu)\cos\frac{\alpha_1}{2}\cos\frac{3\alpha_1}{2}\right],$$

$$S_1(z) \approx \mp\frac{q_1^0}{2\sqrt{\rho_c}}\cos\frac{\alpha_1}{2}\left(1-\frac{1+\nu}{2}\sin\frac{\alpha_1}{2}\sin\frac{3\alpha_1}{2}\right), \tag{7}$$

$$T_1(z) \approx \frac{q_2^0}{4\sqrt{\rho_c}}\cos\frac{\alpha_1}{2}\left(-\nu+\frac{1+\nu}{2}\sin\frac{\alpha_1}{2}\sin\frac{3\alpha_1}{2}\right),$$

$$T_2(z) \approx -\frac{q_2^0}{4\sqrt{\rho_c}}\cos\frac{\alpha_1}{2}\left(1+\frac{1+\nu}{2}\sin\frac{\alpha_1}{2}\sin\frac{3\alpha_1}{2}\right),$$

$$S_1(z) \approx \pm\frac{q_2^0}{4\sqrt{\rho_c}}\sin\frac{\alpha_1}{2}\left[1-\nu-(1+\nu)\cos\frac{\alpha_1}{2}\sin\frac{3\alpha_1}{2}\right].$$

The behavior of the moments in the middle surface of the shell under the action of loads $q_1(\xi)$ and $q_2(\xi)$ in the vicinity of the ends of the segment (a,b) changes when one variant of shell theory is replaced by another. If various variants of the general moment theory of cylindrical shells are used, then the character of the singularities of the moments will be the same as that of the forces; however, the asymptotic formulas may differ for different variants of the theory. If, on the other hand, one proceeds from the theory of shallow shells, then the moments will be bounded. If the shell is loaded by radial forces $q_3(\xi)$ distributed along the segment of the generator (a, b), then the forces T_1, T_2, S_1 and the moments in the middle surface will be bounded.

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REFERENCES

[1] V. M. Darevskii, DAN, **75**, No. 1, 7 (1950).
[2] V. M. Darevskii, in the collection *Strength and Dynamics of Aircraft Engines*, issue 1, Mashinostroenie, 1964, p. 23.
[3] E. I. Grigoluk, V. M. Tolkachev, Proceedings of the 5th All-Union Conference on the Theory of Shells, Moscow, 1965.
[4] A. L. Goldenveizer, *Theory of Elastic Thin Shells*, Moscow, 1953.
[5] F. D. Gakhov, *Boundary-Value Problems*, Moscow, 1958.

Note: Figure translations are in progress. See original paper for figures.