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Abstract

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THE LOCKING BAND OF THE FREQUENCY OF A LASER GENERATOR

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The phenomenon of frequency locking of an ordinary generator was studied in detail by rigorous methods of the theory of oscillations long ago ([1]). This same phenomenon has also repeatedly been considered by means of non-rigorous methods (see, for example, ([2])); such a consideration gives the correct value of the locking band for the case when the external emf is so small that the amplitude of the oscillation in the generator circuit, even when the frequencies of the external emf and of the generator coincide, only slightly exceeds the amplitude that occurs when the external emf is switched off.

Laser generators constitute an auto-oscillatory system in which not only the overall dimensions but also the "active" element have a length considerably exceeding the length of the generated wave. A rigorous analysis of the oscillatory processes in such systems cannot at present be carried out. It is therefore natural to apply a simplified approach to the solution of certain practical problems. The consideration given below has made it possible to determine the frequency locking band of a continuously operating laser generator; the analysis may evidently be regarded as valid for the case of a sufficiently small external force (and consequently also a small locking band).

Let us consider the scheme of a traveling-wave laser generator. For definiteness we shall assume (see Fig. 1) that it contains three reflecting mirrors 1, 2, 3 and one active element 4. The generated wave traverses a closed path, shown by the dotted line. We shall assume that there is only one wave, traveling counterclockwise (in practice this can be obtained by means of a valve). At least one of the mirrors must be partially transmitting in order to make possible the input of the external force. In the figure a partial output of radiation from the generator and the introduction of the external force through mirror 1 are shown.

We shall characterize the active element purely phenomenologically: a stationary wave at its input with amplitude U gives at its output a wave with amplitude $U_1 = \Phi(U)$. Of course, U_1 is a function not only of U, but also of the wave frequency ω; however, since we shall be interested only in a very small interval of frequency variation (the locking band is small!), this dependence may be neglected.

Fig. 1

Figure 1: Fig. 1

First set the external force equal to zero and consider the boundary conditions on the surface of mirror 1. Let us write the wave leaving the mirror in the form $Ue^{j(\omega t-kx-\varphi)}$, where x is the coordinate along the ray path measured from the surface of the mirror. Then the wave returning to the same point of the surface after making the complete circuit may be written in the form

$$\Phi(U)e^{-\beta_1+j(\omega t-kL-\varphi)}, \tag{1}$$

where the factor $e^{-\beta_1}$ takes into account the attenuation of the wave amplitude along its path, caused by various reasons, and L is the optical length of the entire path.* In general, the quantity L is a function of ω owing to the speci-

* In this notation we neglect the change in the wave amplitude along the path from $x=0$ to the active element.

···of the properties of the active element; this leads, in particular, to the fact that the frequency of the generator differs slightly from the frequency of the corresponding conservative system. However, for the question of interest to us, taking this circumstance into account is inessential, and therefore we shall neglect the dependence of L on ω.

Let us denote the reflection coefficient of mirror 1 (in amplitude) by ρ ($\rho \lesssim 1$). Then, from the boundary condition (equality of the tangential components of the electric field), we obtain for the stationary regime the relation

$$U+\rho e^{-\beta_1}\Phi(U)e^{-jkL}=0. \tag{2}$$

Fig. 1

Let us denote $\rho e^{-\beta_1}=e^{-\beta}$. This coefficient takes into account the attenuation of the wave amplitude over a complete cycle of its passage inside the resonator (more precisely, excluding the section occupied by the active element), i.e., over the time $t_1 \approx L/c$, where c is the speed of light. It is easy to see that $\beta \approx \omega t_1/2Q$, where Q is the Q-factor of the resonator; usually $\beta \ll 1$.

It follows from (2) that the self-oscillation frequency ω_0 is determined by the equality

$$k_0L=\frac{\omega_0}{c}L=2\pi n+\pi \tag{3}$$

(n is an integer), and the amplitude of the self-oscillations U_0 by the relation

$$U_0 e^{\beta} = \Phi(U_0). \tag{4}$$

Considering possible sufficiently small deviations of U from U_0, it is easy to show that the stability ("in the small") of the regime with amplitude U_0 requires the fulfillment of the inequality

$$\gamma = (d\Phi/dU)_{U=U_0} < e^{\beta}. \tag{5}$$

Let an external force with frequency ω, very close to ω_0, now act on our system. For simplicity, suppose that the external beam of radiation (see Fig. 1) creates on the inner surface of mirror 1 the same distribution of amplitudes and phases of oscillations as the self-oscillation of the system considered above. In this case the external beam is sufficiently characterized by the amplitude E. If the entrainment phenomenon takes place, then in the system there will be a single wave process with the frequency of the external force ω. For the amplitude of the wave inside the resonator at $x = 0$ we shall retain the previous expression $Ue^{-j\varphi}$. In now forming the boundary condition for the surface of mirror 1, it is necessary to add a term due to the presence of the external force, which we shall write in the form $E\tau e^{j\omega t}$, where τ is the transmission coefficient of the mirror (in amplitude). Instead of (2) we obtain the relation

$$Ue^{-j\varphi} + \Phi(U)e^{-\beta - j(kL+\varphi)} = E\tau. \tag{6}$$

Putting here

$$kL = k_0 L + \varepsilon = 2\pi n + \pi + \varepsilon, \quad \varepsilon \ll 1, \tag{7}$$

and equating the squares of the moduli of both sides of the equality, we obtain

$$U^2 + \left[\Phi(U)e^{-\beta}\right]^2 - 2U\Phi(U)e^{-\beta}\cos\varepsilon = E^2\tau^2. \tag{8}$$

It was noted above that our consideration may be regarded as valid for a sufficiently small external force, when U changes only insignificantly···

exceeds U_0. Denoting $U = U_0 + u$ and assuming $u \ll U_0$, $\cos\varepsilon = 1 - \varepsilon^2/2$, it is easy to obtain approximate expressions for u and the phase shift φ:

$$u = \frac{\sqrt{E^2\tau^2 - \varepsilon^2 U_0^2}}{1 - \gamma e^{-\beta}}, \qquad \operatorname{tg}\varphi = \frac{\varepsilon U_0}{\sqrt{E^2\tau^2 - \varepsilon^2 U_0^2}}. \tag{9}$$

Of greatest interest is the determination of the locking band. The boundaries of this band correspond to the maximum value of $|\varepsilon|$. From (8) it is seen that $|\varepsilon|_{\max}$ occurs for $U = \Phi(U)e^{-\beta}$; in this case $U = U_0$, $u = 0$, $\varphi = \pi/2$, and

$$|\varepsilon|_{\max} = \frac{E\tau}{U_0}. \tag{10}$$

Using (7), we find for the locking band $\delta\omega$ the relation

$$\frac{\delta\omega}{\omega_0} = \frac{2|\omega - \omega_0|_{\max}}{\omega_0} = \frac{2c}{\omega_0 L}|\varepsilon|_{\max} = \frac{\lambda}{\pi L}\frac{E\tau}{U_0}, \tag{11}$$

where λ is the wavelength of the generator. For practical purposes it is expedient to pass to radiation intensities. Denoting the wave intensity in the generator by $P_0 \sim U_0^2$, the wave intensity of the external source by $P \sim E^2$, and introducing $T = \tau^2$, the transmission coefficient of the mirror in power, we write (11) in the form

$$\frac{\delta\omega}{\omega_0} = \frac{\lambda}{\pi L}\sqrt{\frac{PT}{P_0}}. \tag{12}$$

Finally, the intensity P_0 is easily measured from the radiation emerging from the same mirror, whose intensity is $P_b = P_0 T$. We may write

$$\frac{\delta\omega}{\omega_0} = \frac{\lambda}{\pi L}\sqrt{\frac{P}{P_b}}\,T. \tag{13}$$

The formula for the locking band has been obtained by us under the assumption that there is only one traveling wave; the question arises as to what will happen in the presence of two waves traveling toward one another. Of course, in this case the characteristic of the active element $\Phi(U)$ may change substantially; however, since its specific form did not enter the final formula, one may assume that it remains valid in this case as well. For the same reasons one may assume that the result will also be valid for an ordinary laser generator with a standing wave in its resonator. Usually a laser generator simultaneously generates oscillations of several modes. Apparently, this circumstance also cannot noticeably disturb the result obtained, provided that the frequency difference between the modes considerably exceeds the locking band.

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Note: Figure translations are in progress. See original paper for figures.