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Abstract

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AN ANALYTICAL INVESTIGATION OF THE RELATIONS BETWEEN WIND AND CURRENT IN THE EQUATORIAL ZONE OF THE OCEAN

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Let us consider the problem of a current excited by wind in an ocean bounded by the sea surface $z = \zeta(x, y)$, by the surface $z = H = \text{const}$, and by the planes $x = 0$, $x = L = \text{const}$. The origin of coordinates is located on the undisturbed sea surface $z = 0$; the x-axis is directed along the equator to the east, y to the north, and z vertically downward.

We shall regard the current as steady and the fluid as homogeneous. If horizontal exchange of momentum and nonlinear inertial terms are neglected, and if it is taken into account that ζ is small in comparison with H, then we are within the framework of the classical theory of sea currents ([1–4]). Let the coefficient of vertical exchange A be constant, and let the Coriolis parameter be $\Omega = \beta y$, where $\beta = 2\omega/R$, R is the radius of the Earth. We restrict ourselves to the case of a zonal wind, taking the components of the tangential wind stress to be $T_x = T_x(y)$, $T_y = 0$. The basic relations of the theory take the form ([3,4])

$$\vartheta'\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) + \frac{d\lambda'}{dy}\frac{\partial\psi}{\partial x} + \frac{d\vartheta'}{dy}\frac{\partial\psi}{\partial y} = -\frac{dm'T_x}{dy}; \tag{1}$$

$$u = (N - \theta m' - \Lambda n')T_x + (\Lambda\vartheta' - \theta\lambda')\frac{\partial\psi}{\partial x} - (\theta\vartheta' + \Lambda\lambda')\frac{\partial\psi}{\partial y};$$

$$v = (-M + \Lambda m' - \theta n')T_x + (\Lambda\lambda' + \theta\vartheta')\frac{\partial\psi}{\partial x} + (\Lambda\vartheta' - \theta\lambda')\frac{\partial\psi}{\partial y}, \tag{2}$$

$$\frac{\partial\zeta}{\partial x} = -m'T_x - \lambda'\frac{\partial\psi}{\partial x} - \vartheta'\frac{\partial\psi}{\partial y}, \qquad \frac{\partial\zeta}{\partial y} = -n'T_x + \vartheta'\frac{\partial\psi}{\partial x} - \lambda'\frac{\partial\psi}{\partial y}. \tag{3}$$

In (1), (2), (3), u, v are the horizontal components of the current velocity, and ψ is the stream function of the total flows. The coefficients m', n', λ', ϑ' are known functions of the dimensionless argument aH, where $a = \sqrt{\rho\Omega/2A}$. The quantities M, N, Λ, θ are likewise known functions of aH and, in addition, of the coordinate z ([4]).

We shall assume that the discharge of water between the planes $x = 0$ and $x = L$ is zero (in the ocean, under steady circulation, there can be no accumulation of water in one of its parts, for example in the northern one). Then the condition for the function ψ may be written in the form

$$(\psi)_{x=0} = (\psi)_{x=L} = 0. \tag{4}$$

The coefficients m', ϑ' in equation (1) are even, and λ' is an odd function of y. Consequently, they can be expanded respectively in even and odd powers of y. We shall restrict ourselves in the expansions of these coefficients to the linear terms, taking $\vartheta' = 3A/g\rho H^3$, $m' = 3/2g\rho H$, $\lambda' = 6\beta y/5gH$. Instead of (1) we obtain the equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{2\rho\beta H^2}{5A}\frac{\partial\psi}{\partial x} = -\frac{H^2}{2A}\frac{dT_x}{dy}, \tag{5}$$

which, in accuracy up to the coefficients, coincides with equations obtained earlier ([5,6]). Let us prescribe the wind distribution according to the law

$$T_x = -T_0\left[1 - \varepsilon\sin\frac{\pi}{2b}(y + y_0)\right],$$

which describes sufficiently well the variation of the zonal wind stress with latitude in the equatorial zone of the ocean ([7–9]).

The solution of equation (5) with boundary condition (4) has the form

$$\psi = \frac{b}{\pi}\,\varepsilon T_0\,\frac{H^2}{A}\,\Phi(x)\cos\frac{\pi}{2b}(y + y_0), \tag{6}$$

where

$$\Phi(x) = 1 - \frac{e^{k_2L} - 1}{e^{k_2L} - e^{k_1L}}e^{k_1x} - \frac{1 - e^{k_1L}}{e^{k_2L} - e^{k_1L}}e^{k_2x},$$

$$k_{1,2} = -\rho\beta H^2/5A \pm \sqrt{(\rho\beta H^2/5A)^2 + \pi^2/4b^2}.$$

As is seen from (6), for $y = -y_0 \pm b$ the function ψ vanishes. Thus, near the equator a certain closed region is singled out, on the boundary of which this function is known, and we obtain the possibility of studying the flow in the equatorial region of the ocean without considering the motion of the fluid outside it. The approximate solution obtained evidently gives satisfactory results, since we restrict ourselves to studying the motion only near the equator.

Fig. 1. Isolines of the function $\psi \cdot 10^{-6}$ (m^3/sec) and topography of the sea surface ζ (cm) for $y_0 = 0$

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The vertical component of the current velocity w and the level ζ are calculated from the known formulas (4).

The numerical calculations were carried out for $H = 200$ m, $b = 500$ km, $L = 5000$ km (Atlantic Ocean), $A = 50$ cm^2/sec, $T_0 = 0.5$ dyn/cm^2 (wind speed approximately 4 cm/sec), $\varepsilon = 0.8$, and for $y_0 = 0$ and $y_0 \neq 0$.

For the value of the parameter $y_0 = 0$, maps of isolines of the functions ψ and ζ were constructed, as well as maps of currents at the sea surface $z = 0$ and at the depth $z = 2H/3$ (Figs. 1 and 2). As is seen from these figures, the patterns of currents on individual horizons are not at all similar to the distribution of the total transports. If the total transports form a closed circulation similar to that obtained by Munk ([8]), then neither at the sea surface nor at the depth $z = 2H/3$ do we observe it.

At the ocean surface an inter-trade-wind countercurrent is clearly visible ([10]). With a different wind nonuniformity this countercurrent at the ocean surface may also be absent: it will be observed at depth. The condition for its emergence at the surface for $y = b$, $z = 0$ has the form $\varepsilon > (1 + 6.8\Phi)^{-1}$. For fixed ε, the possibility of the countercurrent emerging at the surface increases with increasing ocean width L, since in this case the value of the function Φ increases.

As a consequence of the β-effect, an intense northern current (an analogue of the Guiana Current) appears at the ocean surface in its western part. At the western boundary of the ocean, a southern current beginning south of the equator (an analogue of the Brazil Current) is clearly noticeable.

Of greatest interest is the current on the horizon $z = 2H/3$, where the inter-trade-wind countercurrent, continuously, without changing direction, passes into an equatorial deep countercurrent. More precisely, they begin—

join together in the western part of the ocean and then divide into two concentrated jets in the general eastward flow. The first jet can be explained by a displacement of the thermal equator, i.e., by the existence of a calm zone; the second, situated along the equator, apparently by the existence of the geographic equator ([11]). Calculation near the equator shows that the core of the countercurrent is not located strictly along the equator, but shifts southward as it moves eastward.

Fig. 2. Currents: *I*—at the sea surface $z = 0$ and *II*—at the horizon

Fig. 2

Figure 2: Fig. 2

$z = 2H/3$ for $y_0 = 0$

As is seen from Fig. 1, the ocean surface rises toward the west, being deformed at the same time in the transverse direction. The lowest position of the ocean surface is at its eastern boundary, and the highest is at the western boundary south of the equator.

Let us now consider, for $y_0 = 0$, the features of the current and the level at the equator. Putting $y = 0$ in (6), we obtain $S_x = 0$, $S_y = (beT_0H^2/\pi A)\Phi'$, where S_x, S_y are the components of the total transport, and

$$u = -\frac{T_0 H}{4A}\left(1 - \frac{z}{H}\right)\left(1 - \frac{3z}{H}\right), \qquad v = \frac{3beT_0H^2}{2\pi A}\Phi'\left(1 - \frac{z^2}{H^2}\right); \tag{7}$$

$$w = -\frac{T_0 \rho\beta H^3}{240A^2} z\left(1 - \frac{z}{H}\right)\left(2 + 2\frac{z}{H} - 7\frac{z^2}{H^2} + 3\frac{z^3}{H^3}\right); \tag{8}$$

$$\frac{\partial\zeta}{\partial x} = \frac{3T_0}{2g\rho H}, \qquad \frac{\partial\zeta}{\partial y} = \frac{beT_0}{\pi g\rho H}\Phi'. \tag{9}$$

From (9) it is evident that for $y_0 = 0$ the profile of the zonal velocity component is a parabola with its vertex at the depth $z = 2H/3$, where the velocity of the deep countercurrent at the equator is maximal. As is seen from (8), everywhere on the equator there is an upwelling of water. For $T_0 = 0.5$ dyn/cm^2, $H = 200$ m, the increase in level over a length of 1000 km is 3.75 cm.

In the case $y_0 > 0$, for which the latitudinal distribution of the zonal wind is closer to the real one ([7]–[9]), at the equator the zonal component of the total transport S_x will already be greater than, and not equal to, zero. This leads to an intensification of the deep equatorial countercurrent. The current system remains basically the same as for $y_0 = 0$, only the deep equatorial countercurrent becomes stronger and wider. The zonal component of the velocity at the equator is calculated in this case by the formula

$$u = -\frac{T_0 H}{4A}\left(1 - \frac{z}{H}\right)\left[\left(1 - \varepsilon\sin\frac{\pi y_0}{2b}\right)\left(1 - \frac{3z}{H}\right) - 3\varepsilon\left(1 + \frac{z}{H}\right)\Phi\sin\frac{\pi y_0}{2b}\right], \tag{10}$$

from which it is clear that if the surface current is directed westward, then the ratio of the maximum velocity of the countercurrent to the velocity of the surface current will always be greater than 1/3, which is the case for $y_0 = 0$. Thus,

Fig. 3. Diagrams of the zonal component of velocity at the equator: a — $y_0 = 0,\ A = 50\ \text{cm}^2/\text{sec}$; b —$y_0 = 0.38b,\ A = 50\ \text{cm}^2/\text{sec}$; v — $y_0 = 0.38b,\ A = 10\ \text{cm}^2/\text{sec}$

Figure 3: Fig. 3. Diagrams of the zonal component of velocity at the equator: a —$y_0 = 0,\ A = 50\ \text{cm}^2/\text{sec}$; b —$y_0 = 0.38b,\ A = 50\ \text{cm}^2/\text{sec}$; v —$y_0 = 0.38b,\ A = 10\ \text{cm}^2/\text{sec}$

in the midsection of the ocean at the equator $[x = L/2,\ \Phi(L/2) = 0.3]$ for $y_0 = b/3$ this ratio is equal to 1.8, and when y_0 is increased to $y_0 = 0.38b$ it rises to 3. Thus, with a distribution of wind stress closer to that observed, we obtain a deep equatorial undercurrent intensified in the vertical (Fig. 3). It can be shown that for $y_0 < 0$ the deep equatorial undercurrent will still exist, but will become weaker and narrower than for $y_0 = 0$ or for $y_0 > 0$.

Fig. 3. Diagrams of the zonal component of velocity at the equator:
a —$y_0 = 0,\ A = 50\ \text{cm}^2/\text{sec}$; b —$y_0 = 0.38b,\ A = 50\ \text{cm}^2/\text{sec}$; v —$y_0 = 0.38b,\ A = 10\ \text{cm}^2/\text{sec}$.

It should be noted that for $A = 50\ \text{cm}^2/\text{sec}$ we obtain values of the velocity of the deep undercurrent at the equator that are considerably smaller than those observed ([13,14]). More realistic values are obtained for A = 10-15 cm^2/sec ([11,12]). Thus, for $A = 10\ \text{cm}^2/\text{sec}$ and $\varepsilon\Phi \sin(\pi y_0/2b) = 0.135$ (Fig. 3), at the equator the velocity of the surface current is 36.25 cm/sec (westward current) and the maximum velocity of the deep undercurrent is 110 cm/sec (eastward current).

Thus, the existence of a deep equatorial undercurrent does not depend on the distribution of the wind stress. The best results are obtained when a distribution of wind stress with a positive parameter y_0 is chosen. But for any values of this parameter the intertrade undercurrent and the subsurface equatorial undercurrent exist and constitute two concentrated jets in the general eastward flow. Thus, we do not obtain the fact known from observations that between the undercurrents there is a westward current. However, such motion is obtained if one assumes that at the equator there is a local calm zone, taking, for example,*
T_x

$$= -T_0 \left(1 - \varepsilon \cos \frac{2\pi}{b} y\right)$$

and obtaining

$$S_x = \varepsilon T_0 \frac{H^2}{2A} X(x) \cos \frac{2\pi}{b} y.$$

It is clear from this that in the interval $(-b/4,\ b/4)$ there is an undercurrent intensified both horizontally and vertically, which may be deep, but may also reach the ocean surface; moreover, this equatorial undercurrent is separated from the intertrade undercurrent (at least to some depth) by a westward flow. It is interesting to note that in the interval $(-3b/4,\ -5b/4)$ the motion is again eastward, i.e., there is an undercurrent south of the equator. This fact is confirmed by observations in the Pacific Ocean, where the corresponding undercurrent was recently discovered ([16]).

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* This law approximately describes the distribution of wind stress obtained from observations in the Pacific Ocean ([15]).

Note: Figure translations are in progress. See original paper for figures.