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AERODYNAMICS

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Abstract

Full Text

AERODYNAMICS

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ON THE EQUATIONS OF AERODYNAMICS IN THE PRESENCE OF BINARY MOLECULAR PROCESSES

(Presented by Academician A. A. Dorodnitsyn, March 19, 1965)

In recent years attention has been devoted to the study of various processes in the flow of bodies by a gas stream with excited internal degrees of freedom. For such flows, methods for obtaining equations and determining dissipation are very topical. The use of phenomenological methods for this purpose, in particular the thermodynamics of irreversible processes, is unsatisfactory, first, because of the ambiguity of the equations obtained and, second, because of the requirement of small deviation from complete thermodynamic equilibrium.

These shortcomings can be eliminated if hydrodynamic systems are constructed on the basis of the equations of kinetics. With this method, in order to obtain the unique form of the equations of aerodynamics, it is not necessary to know the exact character of the interaction of molecules in the gas, i.e., the scattering cross sections and transition probabilities; only information about their order of magnitude is required. Exact knowledge of collision cross sections is necessary in calculating kinetic coefficients. Of course, the presence of different types of interaction in a gas leads to different hydrodynamic systems of equations. Thus, for example, if l_t, l_i are the lengths for establishing equilibrium in the translational and internal degrees of freedom, l_{ti} is the length of the relaxation zone, and L is the characteristic size of the body, then for $l_t \sim l_i \sim l_{ti} \ll L$ we obtain the class of quasi-equilibrium flows, which at present has been studied in sufficient detail. For $l_t \ll l_i \sim l_{ti} \sim L$ we have the so-called "relaxation of the mixture," and for $l_t \sim l_i \ll l_{ti} \sim L$, "two-temperature" relaxation, which is investigated below up to the calculation of the kinetic coefficients. A special class of equations of relaxation aerodynamics for vibrational multiquantum relaxation and nonequilibrium radiation, belonging to the latter case, was analyzed in ([1,2]).

The general methodology of these works is also suitable for other degrees of freedom, in particular for dissociation, if not only binary but also triple collisions are taken into account in the proper way. It is also not very difficult to take into account rotations, or the simultaneous presence of vibrations and rotations. However, owing to the absence of sufficiently accurate and reliable data on collision diameters for rotations, this degree of freedom is not considered in detail. The process of obtaining equations for vibrational relaxation at

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a strong deviation from the state of equilibrium $(T - T_i)/T \sim 1$ can be briefly described as follows ([1]). The collision term of the Boltzmann equation is split into parts in accordance with the different collision cross sections (fast and slow processes), as a result of which, in the dimensionless form of the kinetic equation, a large parameter appears in one part of the collision terms. This leads to the appearance of new summational invariants and, consequently, additional hydrodynamic equations. In case ([1]) the system of linearly independent invariants has the form $1, mc, mc^2/2, E_N$, and the equilibrium distribution function

$$f^{(0)} = n\left(\frac{m}{2\pi kT}\right)^{3/2}\left[\sum_{(N)}\exp\left(-\frac{E_N}{kT_i}\right)\right]^{-1}\exp\left(-\frac{mC^2}{2kT}-\frac{E_N}{kT_i}\right). \qquad (1)$$

Everywhere below we use the generally accepted notation of kinetic theory ([3]). The hydrodynamic system in the first approximation may be written in the form

$$\frac{dn}{dt}+n\frac{\partial c_0}{\partial r}=0, \qquad nm\frac{dc_0}{dr}=-\frac{\partial p}{\partial r},$$
$$\frac{d\overline{E}}{dt}+kT\frac{\partial c_0}{\partial r}=0, \qquad \frac{dE_i(T_i)}{dt}=\overline{E_N\Sigma'}. \qquad (2)$$

Here

$$n\overline{E_N\Sigma'}=\sum_{(N)}{}' E_N\int\left(f^{(0)'}f_1^{(0)'}-f^{(0)}f_1^{(0)}\right)d\mathcal{P}\,dc\,dc_1. \qquad (3)$$

Σ' denotes summation over the set of inelastic collisions, $p = nkT$,

$$\overline{E}=NkT/2+E_i(T_i), \qquad E_i(T_i)=\sum_{(N)}E_Ne^{-E_N/kT_i}\Big/\sum_{(N)}e^{-E_N/kT_i}, \qquad d\mathcal{P}=\beta g b\,db\,d\varepsilon.$$

As shown in ([4]), equation (3) for a harmonic oscillator with an infinite number of excitation levels reduces to the classical relaxation equation of Landau–Teller type. For a truncated oscillator this equation has a more complicated form and explicitly depends on the internal temperature T_i.

The corresponding relaxation times were calculated on an electronic computer ([4]) using cross sections ([5]) and two experimentally determined quantities: $\theta_0 = h\nu/k$ from spectroscopic data and the characteristics of the Lennard-Jones potential from viscosity experiments. The data obtained (Fig. 1) agree very well with the experimental values of τ of various authors over a wide temperature range, which indicates that the harmonic-oscillator model is a fairly good approximation to reality.

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Fig. 1. Experimental data:
a –Horiuchi; b –Strelova and Kozak; v –Blekman; g –Shilling and Portington; d –Kantrowitz and Huber; e –Lukashik and Young. Curves –calculation.

The structural solution for the distribution function in the second approximation may be represented in the form

$$\Phi^{(1)} = A^{(1)}\frac{\partial T}{\partial r} + A^{(2)}\frac{\partial T_i}{\partial r} + B : \frac{\partial c}{\partial r} + A_3. \tag{4}$$

The unknown vectors $A^{(1)}$, $A^{(2)}$ and tensor B satisfy the integral equations

$$f^{(0)}\left(\frac{mC^2}{2kT} - \frac{5}{2}\right)C = \sum_{(N)}\int f^{(0)}f_1^{(0)}\left(A^{(1)\prime} + A_1^{(1)\prime} - A^{(1)} - A_1^{(1)}\right)d\mathcal{P}\,dc,$$

$$f^{(0)}\frac{(E_N - E_i)}{kT_i}C = \sum_{(N)}{}'\int f^{(0)}f_1^{(0)}\left(A^{(2)\prime} + A_1^{(2)\prime} - A^{(2)} - A_1^{(2)}\right)d\mathcal{P}\,dc, \tag{5}$$

$$-2f^{(0)}\overline{CC} = \sum_{(N)}\int f^{(0)}f_1^{(0)}\left(B' + B_1' - B - B_1\right)d\mathcal{P}\,dc.$$

Although all the subsequent arguments pertain to a diatomic gas (6), they also generalize to the case of a mixture (7). Equations (5) are solved by the Ritz method. In this case $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$, $\mathbf{B}$ are expanded into finite systems of series in orthogonal polynomials. Some difficulty is presented by the representation of E_N, for which the Sonine polynomials commonly used are unsuitable, owing to the discrete, i.e., quantum-mechanical, description of the inertial degrees of freedom. For the internal energy the polynomials proposed in (8) prove extremely useful:

$$P_N^{(p)}\left(\frac{E_N}{kT_i}\right) = -\frac{E_N P_N^{(p-1)}}{kT_i} + \sum_{q=0}^{p-1}\left[\frac{E_N P_N^{(p-1)}P_N^{(q)}}{kT_i}\Big/\overline{P_N^{(q)2}}\right]P_N^{(q)}. \tag{6}$$

Let us note that if expansions of type (6) with respect to the space of internal energy are not used, it is impossible to construct a rigorous method of successive approximations (9). At the temperature T_i the expansions for $\mathbf{A}^{(1)}$, $\mathbf{A}^{(2)}$, $\mathbf{B}$ have the form

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$$\mathbf{A}^{(i)} = C\sum_{r}^{\xi-1}\sum_{p}^{\eta-1} a_{rp}^{(i)} S_{3/2}^{(r)}\left(\frac{mC^2}{2kT}\right) P_N^{(p)}\left(\frac{E_N}{kT_i}\right),$$
$$\mathbf{B} = \overline{CC}\sum_{r}^{\xi-1}\sum_{p}^{\eta-1} b_{rp} S_{5/2}^{(r)}\left(\frac{mC^2}{2kT}\right) P_N^{(p)}\left(\frac{E_N}{kT_i}\right). \tag{7}$$

The desired dissipative coefficients can be expressed through $a_{rp}^{(i)}$, b_{rp}

$$\lambda_t = -\frac{5}{2}\frac{pk}{m}a_{10}^{(1)}, \qquad \lambda_k = -\frac{p}{m}c_v a_{01}^{(2)}, \qquad \mu = \frac{kT}{2}b_{00},$$
$$\lambda_{tk} = -\frac{T_i}{T}\frac{p}{m}c_v a_{01}^{(1)}, \qquad \lambda_{kt} = -\frac{T}{T_i}\frac{5}{2}\frac{pk}{m}a_{10}^{(2)}. \tag{8}$$

Carrying out the necessary operations by the Ritz method, we replace (5) by an algebraic system of the form

$$\sum_{s,q}^{\xi-1} G^{rpsq} t_{sq}^{(j)} = -R_{rp}, \tag{9}$$

where

$$R_{rp} = \sum_{(N)}\int FWS_m^{(r)}P_N^{(p)}\,dc,$$

where F denotes the left-hand sides of (5); $W = \vec{\xi}$ for λ and $W = \overline{\vec{\xi}\vec{\xi}}$ for μ. The reduced velocity is $\vec{\xi} = \sqrt{m/2kT}\,C$.

The expression

$$G^{rpsq} = n^2\{[WQ^{rp}; WQ^{sq}] + [WQ^{rp}; W_1Q^{sq}]\}, \tag{10}$$

where

$$[WQ^{rp}; W_1Q^{sq}] = \frac{1}{n^2}\sum_{N}\int f^{(0)}f_1^{(0)}(WQ^{rp} - W'Q^{rp})W_1Q^{sq}\,d\mathcal{P}\,dC\,dC_1, \tag{11}$$

where $Q^{rp} = S_m^{(r)}P_N^{(p)}$.

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Fig. 2: graph of $\lg \lambda_k \cdot 10^7$ versus $\lg T/10^3$, with curves for O_2 and N_2, and labels λ_t, λ_k, $T_k = T$, $T_k = 0.1T$.

Figure 1: Fig. 2: graph of $\lg \lambda_k \cdot 10^7$ versus $\lg T/10^3$, with curves for O_2 and N_2, and labels λ_t, λ_k, $T_k = T$, $T_k = 0.1T$.

Thus, the kinetic coefficients can be computed using the solution of system (9), if the necessary collision cross sections are substituted into (11). For the harmonic-oscillator model (5), such calculations have been carried out.

From the results of the calculation of the dissipative coefficients one can draw the following conclusions: viscosity, diffusion, and thermal diffusion (7) have the same form as in an unexcited gas. The thermal-diffusion coefficient due to the gradient $\partial T_i/\partial r$ vanishes. The same is obtained for the "cross" thermal conductivity, i.e. $\lambda_{kT} = \lambda_{tk} = 0$. The latter

The result was also checked for the third approximation to the expansions $A^{(1)}$, $A^{(2)}$, in which λ_t, λ_k receive small corrections. Of fundamental importance for the effect on thermal conductivity is the presence of resonant transitions. It is precisely this type of interaction that makes it possible to introduce rigorously the internal temperature T_i in accordance with the general principles of the kinetic theory of gases. In this sense the introduction of T_i in (10) is insufficiently justified. Without taking resonant exchanges into account, we obtain formulas for λ_t, λ_k that, for $T = T_i$, coincide with (11), in the case where there is no energy exchange between translational and internal degrees of freedom.

Fig. 2

Taking into account the resonant mechanism of heat transfer by quanta and for $T = T_i$, we can obtain an expression for λ_k from (9). For a large departure from the equilibrium state, $(T - T_i)/T \sim 1$, we have

$$\lambda_t = \frac{15}{4}\frac{\mu k}{m}, \quad \lambda_k = nDc_v(T) \times \left[1 - \frac{1}{\pi^2}\left(\frac{\alpha}{v}\right)^2 \frac{kT}{m}\right]^{-1}, \tag{12}$$

where D is the coefficient of self-diffusion, and α is the constant of the interaction potential. Calculations carried out for O_2, N_2, Cl_2, J_2 show a strong influence of nonequilibrium on the magnitude of the heat flux. In Fig. 2 data are presented for O_2 and N_2, taking into account the contribution of resonant exchanges to the flux λ_k. Quantitatively, in comparison with the diffusive mechanism, the contribution of resonant transitions is small, of the order of 5-10%. What is significant is the general influence of nonequilibrium, i.e., of the difference between T and T_i, on the ratio $\lambda_k/(\lambda_t + \lambda_k)$. From Fig. 2 one may conclude that, although the vibrational degree of freedom is relatively low in energy capacity (in comparison, for example, with dissociation), in fractions of the change of the quantity E_i, λ_k may change by its own order of magnitude. A similar fact evidently also occurs in more general cases, which must be taken

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into account both in calculating the structure of shock waves and in flows of the boundary-layer type.

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REFERENCES CITED

1. V. N. Zhigulev, *Inzh. zhurn.*, **3**, issue 1, 137 (1963).
2. V. N. Zhigulev, *Inzh. zhurn.*, **4**, issue 2, 231 (1964); issue 3, 431 (1964).
3. S. Chapman, T. Cowling, *The Mathematical Theory of Non-uniform Gases*, IL, 1960.
4. B. V. Egorov, *Izv. AN SSSR, Mekhanika*, No. 3, 175 (1965).
5. W. J. Wittman, *J. Chem. Phys.*, **35**, No. 1 (1961).
6. V. M. Kuznetsov, *Izv. AN SSSR, Mekhanika*, No. 3, 178 (1965).
7. V. M. Kuznetsov, *Inzh. zhurn.*, **5**, No. 5 (1965).
8. L. Waldman, E. Trübenbacher, *Zs. Naturforsch.*, **17a**, H. 5, 363 (1962).
9. E. V. Samuilov, *Physical Gas Dynamics*, Moscow, 1959, p. 59.
10. T. E. Morse, *Phys. Fluids*, **7**, No. 2, 159 (1964).
11. J. Hirschfelder, C. Curtiss, R. Bird, *Molecular Theory of Gases and Liquids*, IL, 1961.

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