ON THE CALCULATION OF PROCESSES CAUSED BY THE ENTRY OF ICE CRYSTALS INTO SUPERCOOLED FINE-DROPLET CLOUDS

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Abstract

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ON THE CALCULATION OF PROCESSES CAUSED BY THE ENTRY OF ICE CRYSTALS INTO SUPERCOOLED FINE-DROPLET CLOUDS

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Among the systems of equations proposed for calculating cloudiness, there are systems whose solutions may describe snowfall ([1,2]). Unfortunately, some of the functions entering into these equations, which must be specified, have not been found, and the solutions of these systems cannot yet be carried through numerically.

Let us set ourselves the goal of finding such a system of equations whose solution could give the specific humidities of the gaseous (q_1), liquid (q_2), and solid (q_3) phases of water as functions of spatial coordinates and time in a cloud and below it, under natural processes and under artificial action on the cloud. At the same time it must be possible to carry the solution of the system through numerically; i.e., all functions and parameters entering the equations, except for the unknowns, must be known. The system must describe processes in a cloud containing crystals and small supercooled water droplets; it must describe snowfall. The equations must be applicable in that region of negative temperatures in which spontaneous formation of ice crystals from droplets or from vapor does not occur. In addition to equations for calculating q_1, q_2, q_3, the system will include the heat-inflow equation and equations for determining the numbers of droplets (N_2) and crystals (N_3) per unit mass of air. The calculations will be carried out under the following assumptions: 1) water droplets are completely carried along by the air; 2) crystals have the form of spheres; 3) two particles coagulate only if they belong to different phases; 4) the collection coefficient of droplets by crystals is equal to unity; 5) the particle spectrum of each of the phases is described by a function whose general form does not change (only the values of the parameters change); 6) at the initial moment cloudiness is present (even if with very small thickness and water content). We shall assume that outside the cloud the number of water droplets is equal to the number of condensation nuclei, but that these droplets have radii equal to zero.

To solve the stated problem it is necessary to find the following quantities, referred to unit mass of air and unit time: 1) the mass f_1 of vapor condensing on droplets (for $f_1 > 0$ vapor condensation occurs; for $f_1 < 0$, evaporation of droplets); 2) the mass f_2 of vapor converted into ice during sublimation on crystals (for $f_2 > 0$ vapor sublimation occurs; for $f_2 < 0$, evaporation of crystals); 3) the mass f_3 of water in droplets deposited on crystals and freezing there; 4) the change in the mass f_4 of ice due to the gravitational fall of crystals; 5) the change in the number of droplets n_1 upon their coagulation with crystals; 6) the change in the number of crystals n_2 due to their gravitational fall; 7) the number of crystals n_3 arising from vapor under the action. The mean mass μ of a crystal arising under the action is known ([3]). The mass of refrigerant m evaporating per unit mass of air per unit time varies from one case of action to another and must be specified each time. We introduce the notation: t —time; u_1, u_2, u_3 —the velocities of the air in the direc-

in corresponding rectangular Cartesian coordinates x_1, x_2, x_3 (x_3 is vertical, with origin at ground level and directed upward); k_1, k_2, k_3 are the turbulence coefficients along x_1, x_2, x_3, respectively ($k_1 = k_2$); ρ is density; P is pressure; T is the absolute air temperature; c_p is the specific heat of air at constant pressure; L_1, L_2 are the specific heats of vapor condensation and water freezing, respectively; L_0 is the heat absorbed from the air during evaporation of a unit mass of refrigerant. As follows from the definition of the quantities f_ν ($\nu = 1, 2, 3, 4$), n_j ($j = 1, 2, 3$), μ, m, the equations to be obtained, with turbulence taken into account, will have the form:

$$\frac{dq_1}{dt} = -f_1 - f_2 - \mu n_3 + \frac{1}{\rho}\sum_i \frac{\partial}{\partial x_i}\left(\rho k_i \frac{\partial q_1}{\partial x_i}\right); \tag{1}$$

$$\frac{dq_2}{dt} = f_1 - f_3 + \frac{1}{\rho}\sum_i \frac{\partial}{\partial x_i}\left(\rho k_i \frac{\partial q_2}{\partial x_i}\right); \tag{2}$$

$$\frac{dq_3}{dt} = f_2 + f_3 + f_4 + \mu n_3 + \frac{1}{\rho}\sum_i \frac{\partial}{\partial x_i}\left(\rho k_i \frac{\partial q_3}{\partial x_i}\right); \tag{3}$$

$$\frac{dN_2}{dt} = n_1 + \frac{1}{\rho}\sum_i \frac{\partial}{\partial x_i}\left(\rho k_i \frac{\partial N_2}{\partial x_i}\right); \tag{4}$$

$$\frac{dN_3}{dt} = n_2 + n_3 + \frac{1}{\rho}\sum_i \frac{\partial}{\partial x_i}\left(\rho k_i \frac{\partial N_3}{\partial x_i}\right); \tag{5}$$

$$\frac{dT}{dt} = \frac{1}{c_p}\left[\frac{1}{\rho}\frac{dP}{dt} + L_1 f_1 + (L_1 + L_2)(f_2 + \mu n_3) + L_2 f_3 - L_0 m\right] + \frac{1}{\rho}\sum_i \frac{\partial}{\partial x_i}\left(\rho k_i \frac{\partial T}{\partial x_i}\right). \tag{6}$$

Here

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \sum_i u_i \frac{\partial}{\partial x_i}.$$

In summation, i everywhere takes the values 1, 2, 3. If the system of equations (1)–(6) is supplemented by the equations of motion and the continuity equation, then for the system (1)–(6) u_i and P may be regarded as known. The relation between the quantities P, ρ, T will be determined by the equation of state.

Before calculating the functions f_ν, n_j, let us approximate certain dependences and carry out preliminary calculations. We shall use an approximation, rather than the corresponding differential relations, for the dependence of the saturated-vapor pressure over water (E_2) and over ice (E_3) on temperature. The function approximating $E_2(T)$ is known:

$$E_2 = E_0 \exp \frac{a_1 T - a_2}{T - a_3}, \tag{7}$$

where $E_0 = 6.108$ mb, $a_1 = 17.57$, $a_2 = 4798°$K, $a_3 = 31.1°$K. We approximate the dependence $E_3(T)$ for $T \leq 273°$ by the function

$$E_3 = E_2 - A(T_0 - T)e^{-\alpha(T_0 - T)}. \tag{8}$$

If $A = 6.146 \cdot 10^{-2}$ mb $\cdot$ deg^{-1}, $T_0 = 273°$K, $\alpha = 8.333 \cdot 10^{-2}$ deg^{-1}, then this function agrees sufficiently closely with the tabulated data. The agreement is especially close in the interval $258° < T < 273°$, where the discrepancy for $(E_2 - E_3)$ is less than $0.02(E_2 - E_3)$. We approximate the dependence of the fall velocity v of an individual crystal in still air on the radius r_3 of an ice sphere equal in mass to the crystal by a function whose general form does not differ from the form of the function proposed earlier ([4]) for the fall velocity of a droplet:

$$v = B(1 - e^{-\beta r_3} - \beta r_3 e^{-\chi r_3}) \qquad (0 < \beta < \chi). \tag{9}$$

The maximum fall velocity of snow particles in most snowfalls is about 150 cm $\cdot$ s^{-1} (aggregation of particles does not lead to a substantial increase in velocity); at $r_3 \sqrt[3]{\gamma_3/\gamma_2} = 2 \cdot 10^{-2}$ cm the fall velocity is close

to 100 cm sec^{-1} (5) (γ_2 is the density of water, γ_3 the density of ice). For $r_3 = 10^{-3}$ cm we shall assume that v is somewhat less than the velocity of a drop of the same radius and is equal to 1 cm sec^{-1}. The parameters corresponding to these data are: $B = 150$ cm sec^{-1}, $\beta = 70$ cm^{-1}, $\varkappa = 140$ cm^{-1}. As the distribution function of both drops and crystals with respect to their radii r_l, we take the Khrgian-Mazin function (6). It is easy to verify that this function, referred to unit mass of air, can be written in the form

$$\Phi_l = \frac{1}{2} N_l b_l^3 r_l^2 e^{-b_l r_l}, \tag{10}$$

where

$$b_l = 2\sqrt[3]{10\pi\gamma_l N_l / q_l}. \tag{11}$$

For $l = 2$ all quantities refer to the liquid phase, and for $l = 3$ to the solid phase. Let us find the sum of the particle radii σ_l per unit mass of air; for the system of crystals falling with velocity v, different for each crystal, let us find the mean velocities weighted by the number of particles (η_3), by the areas of their great circles (η_5), and by their mass (η_6). Integration, taking into account (9), (10), gives

$$\sigma_l = \int_0^\infty r_l \Phi_l \, dr_l = \frac{3N_l}{b_l}; \tag{12}$$

$$\eta_\alpha = \int_0^\infty r_3^{\alpha-3} \Phi_3 v \, dr_3 \Big/ \int_0^\infty r_3^{\alpha-3} \Phi_3 \, dr_3 = B\left[1 - \frac{b_3^\alpha}{(b_3+\beta)^\alpha} - \frac{\alpha\beta b_3^\alpha}{(b_3+\varkappa)^{\alpha+1}}\right]. \tag{13}$$

Note that from the first equality in (13) it follows that

$$\int_0^\infty r_3^{\alpha-3} \Phi_3 v \, dr_3 = \eta_\alpha \int_0^\infty r_3^{\alpha-3} \Phi_3 \, dr_3. \tag{14}$$

Let us proceed to the calculation of f_ν, n_j. Denote by f_{1e} the change in the mass of a drop per unit time. Then the known (6) equation for the change in drop mass, in the notation adopted here, will be written as $f_{1e} = 4\pi D\rho(q_1 - 0.622\, E_2/P) r_2$, where D is the diffusion coefficient. The sum of these quantities over all drops in unit mass of air will give f_1. Taking (12) into account, we obtain

$$f_1 = 12\pi D\rho \frac{N_2}{b_2}\left(q_1 - 0.622\frac{E_2}{P}\right). \tag{15}$$

Similarly we obtain

$$f_2 = 12\pi D\rho \frac{N_3}{b_3}\left(q_1 - 0.622\frac{E_3}{P}\right). \tag{16}$$

The function f_3 for one crystal will have the form $f_{3e} = \pi\rho q_2 v r_3^2$. To obtain f_3, multiply this expression by Φ_3 and integrate with respect to r_3 from 0 to ∞, taking first (14) and then (10) into account. Then we shall have

$$f_3 = 12\pi\rho q_2 \frac{N_3}{b_3^2}\eta_5. \tag{17}$$

To calculate f_4, introduce the flux along the x_3 axis of the mass of crystals falling with velocities v:

$$-\frac{4}{3}\pi\rho\gamma_3 \int_0^\infty r_3^3 \Phi_3 v \, dr_3.$$

Differentiation of this expression with respect to x_3 (we assume ρ not to vary with time) and reversal of the sign gives the change in the mass of the solid phase, caused by the gravitational fall of crystals, per unit volume per unit time.

$$\rho f_4 = \frac{\partial}{\partial x_3}\left(\frac{4}{3}\pi\rho\gamma_3 \int_0^\infty r_3^3 \Phi_3 v \, dr_3\right).$$

Taking (14), (10), (11) into account, we obtain:

$$f_4 = \frac{1}{\rho}\frac{\partial}{\partial x_3}\left(\rho q_3 \eta_6\right). \tag{18}$$

We find the function n_1 analogously to f_3. In one unit of time, the number of drops deposited on one crystal will be $-n_{1e} = \pi N_2 v r_3^2$. After the same transformations as were carried out in obtaining f_3, we shall have

$$n_1 = -12\pi\rho N_2 \frac{N_3}{b_3^2}\eta_5. \tag{19}$$

We obtain the function n_2 analogously to f_4, taking into account that the corresponding flux of the number of crystals along the x_3 axis is $-\rho\int_0^\infty \Phi_3 v \, dr_3$:

$$n_2 = \frac{1}{\rho}\frac{\partial}{\partial x_3}(\rho N_3 \eta_3). \tag{20}$$

The quantity n_3 is expressed in terms of the prescribed quantity m and the known [3] quantity—the number of crystals n_0 formed as a result of evaporation of a unit mass of the refrigerant:

$$n_3 = n_0 m. \tag{21}$$

If into the right-hand side of the second equality (13) we introduce b (11), and the expressions obtained thereafter, b (11), E_l (7), (8), are substituted into f_v

(15)–(18), n_1 (19), n_2 (20); if the expressions obtained for f_v, n_1, n_2, and n_3 (21) are substituted into equations (1)–(6) and this system is supplemented by the equation of state, then, for known u_i and P, we obtain a closed system of equations. In these equations all functions (except the unknowns) and parameters are known, and the system can be used for solving specific problems.

The form of the boundary conditions for the equations obtained may vary depending on the particular problem. Let us consider one of the variants of these conditions on the lower and upper boundaries of the domain in which the solution is sought. Let clouds of the lower tier and the crystalline clouds located above, from which small crystals fall out, be separated by a cloudless layer. At the level $x_3 = h$, which lies in this layer and which the lower cloudiness is known not to reach in its development, q_1, q_3, N_3, T are known as functions of the horizontal coordinates and time for the entire period for which the calculation will be made. Let the course of the temperature at the earth, covered with snow, be known for this period, and let the number of condensation nuclei here be known. It is also known that at the level $x_3 = h$ these nuclei are two orders of magnitude fewer. The velocity and pressure fields of the air are also prescribed. It is required to describe the processes in the lower cloudiness both under the natural evolution of the cloudiness ($m = 0$) and when a refrigerant is introduced into the cloud ($m \neq 0$). In solving this problem, the boundary conditions for T, q_1, N_2 at the level $x_3 = 0$ will be: $T = \tau(x_1, x_2, t)$, $q_1 = 0.622E_3(\tau)/P$, $N_2 = C$. Here τ is a prescribed function, C a prescribed constant. Crystals and drops that have reached the ground no longer participate in the processes under consideration. Therefore here one may introduce the conditions: $q_2 = 0,\ q_3 = 0,\ N_3 = 0$. We take the plane $x_3 = h$ as the upper boundary of the domain. The boundary conditions here will be: $q_1 = \bar{q}_1,\ q_2 = 0,\ q_3 = \bar{q}_3,\ N_2 = 10^{-2}C,\ N_3 = \bar{N}_3,\ T = \bar{T}$, where $\bar{q}_1, \bar{q}_3, \bar{N}_3, \bar{T}$ are prescribed functions of x_1, x_2, t.

The system of equations obtained reflects the dependence of the crystallization process of lower-tier clouds on the entry into them of crystals either from upper clouds or due to artificial action; it reflects the microstructure of the cloud and the separation of large particles from small ones during their fall. The method for taking into account these factors, which are very important for the formation of precipitation, as well as the functions f_v, n_j obtained, can also be used in solving problems more general than the one considered here.

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Note: Figure translations are in progress. See original paper for figures.