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Abstract

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Determination of the Imaginary Part of the Phase Shifts of Elastic pp Scattering at 655 MeV

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A phase analysis of elastic pp scattering, taking into account data on meson production ([1,2]), gave several close solutions, equally probable according to the χ^2 criterion. However, only data on the total cross sections of the reactions

$$
\begin{aligned}
&\mathrm{p} + \mathrm{p} \to \pi^+ + \mathrm{p} + \mathrm{n}, \quad &(a)\\
&\mathrm{p} + \mathrm{p} \to \pi^+ + \mathrm{d}, \quad &(b)\\
&\mathrm{p} + \mathrm{p} \to \pi^0 + \mathrm{p} + \mathrm{p}, \quad &(c)
\end{aligned}
$$

were used, which made it possible to find the states of the pp system in which meson production must be taken into account, and the "absorption coefficients" ([3]), averaged over the states $^3P_{0,1,2}$ and $^3F_{2,3}$, and in the $^1D_2$ state.

The elements of the S-matrix of elastic pp scattering in a state with total angular momentum J and orbital angular momentum l are expressed in terms of complex phase shifts

$$S_{Jl} = e^{2i({\delta_{Jl}}^R + i\delta^I_{Jl})}, \tag{1}$$

where δ^R_{Jl} and δ^I_{Jl} are real numbers.

The inelastic-scattering cross section, equivalent at 660 MeV to the cross section for production of a single π-meson, is

$$\sigma^{un}_{Jl}(\mathrm{pp}) = \frac{\pi}{2k^2}(2J+1)(1 - r^2_{Jl}); \tag{2}$$

$$r_{Jl} = e^{-2{\delta_{Jl}}^I}; \tag{3}$$

r_{Jl} is the absorption coefficient in the state (J, l) ([3]), $\vec{k}$ is the wave vector of the incident proton.

With the aid of the resonance model ([4]), developed by Mandelstam, one can find the cross sections for π-meson production from the initial states of the pp system.

In the resonance model ([4]), the matrix elements for the transition from the initial pp state to the final state of a π-meson and two nucleons were assumed not to depend on the meson energy and entered the theory as experimentally determined parameters. The entire energy dependence of the total cross sections and π-meson spectra was determined only by the processes of interaction of the π-meson with a nucleon and the subsequent interaction of the two nucleons in the final state. The nucleon that does not interact with the π-meson is considered only in the S- and P-states relative to the center of mass (c.m.) of the pp system (respectively the S-term and P-term).

From comparison of the angular distributions of π-mesons from reaction (b) at the production threshold and at $E_p = 660$ MeV ([5]), it followed that the reaction proceeds from the $^1D_2$ state with the S-term. Formation of the deuteron with the P-term was neglected.

The differential cross section for meson production from the state (J, l) in the c.m.s. is

$$d^2\sigma/d\omega\, dQ = {}^0\sigma_{Jl}(Q) + {}^2\sigma_{Jl}(Q) P_2(\cos\theta), \tag{4}$$

where Q is the momentum of the π-meson in units $m_\pi = c = 1$; θ is the emission angle of the π-meson in the c.m.s. relative to the collision direction.

The coefficients of the expansion in Legendre polynomials are

$$^0\sigma_{Jl}(Q) = |\beta_{Jl}|^2 a_{Jl}(Q), \tag{5a}$$

$$^2\sigma_{Jl}(Q) = |\beta_{Jl}|^2 b_{Jl}(Q). \tag{5b}$$

The functions of Q: $a^+_{Jl}(Q)$, $a^0_{Jl}(Q)$, $b^+_{Jl}(Q)$, and $b^0_{Jl}(Q)$, which take into account the interaction of the meson with the nucleon and of the nucleon with the nucleon in the final state, the density of final states, and the normalization of the wave functions, are calculated with allowance for conservation of total angular momentum, parity, and isotopic spin. The superscripts refer respectively to the production of π^+- and π^0-mesons.

The total cross section for meson production from the (J, l)-state is

$$\sigma_{Jl} = 4\pi|\beta_{Jl}|^2 \left\{ \int_0^{Q_{\max}} a^+_{Jl}(Q)\, dQ + \int_0^{Q_{\max}{}^0} a^0_{Jl}(Q)\, dQ \right\} = 4\pi|\beta^+_{Jl}|^2 (A_{Jl} + A^0_{Jl}), \tag{6}$$

$Q^0_{\max} \neq Q_{\max}$ because of the difference between the masses of π^0 and π^+; β_{Jl} are the elements of the unitary transition matrix T, which includes possible intermediate processes.

The matrix elements β_{Jl} are related to the transition matrix elements η_{Jl}, which do not take possible intermediate processes into account, by the relations (see formula (5.8) in paper [4])

$$|\beta_{Jl}|^2 = |\eta_{Jl}|^2 \cos^2 \delta^R_{Jl} / \left[1 + \frac{k^2}{4\pi(2J+1)} \overline{\sigma}_{Jl}\right]^2, \qquad (7)$$

where $\overline{\sigma}_{Jl} = 4\pi|\eta_{Jl}|^2(A^+_{Jl} + A^0_{Jl})$ is the "uncorrected" cross section.

If one restricts oneself only to S- and P-waves, then meson production can proceed in the states ${}^3P_{0,1,2}$, ${}^1D_2$, ${}^3F_{2,3}$; nevertheless, in work [4] the ${}^3F_2$- and ${}^3F_3$-states were not taken into account. Using the formalism of the resonance model (Appendix 2 in [4]), one can calculate the corresponding $a_{Jl}(Q)$, $b_{Jl}(Q)$, A_{Jl}, and B_{Jl}, and find the contribution of the ${}^3F_{2,3}$-states to meson production. However, to do this it is necessary to redetermine the model parameters from the experimental data.

To determine the parameters at an incident-proton energy of 655 MeV, measurements were used of the spectrum of π^+-mesons from reactions (a) and (b) at an angle of 30° in the laboratory system ($\theta \approx 54°$ in the c.m.s.). The measurements were carried out with a magnetic spectrometer [6]. The absolute scale of cross sections was calibrated by comparing the proton count at an angle of 56° in the l.s. and the differential cross section of elastic pp-scattering [7]. In the measurement results, in addition to corrections for nuclear absorption and decay of π-mesons, for admixtures of μ-mesons and electrons, a correction was introduced for the loss of μ-mesons relative to the loss of calibration protons due to multiple Coulomb scattering. The correction for multiple scattering took into account both the loss of particles because of their deviation from the allowed trajectories in the spectrometer and the increase in the count due to the broadening of the registered momentum interval $2\Delta p$. Since the π^+-meson spectrum in the laboratory system began at $p > 200$ MeV/c, the admixture of μ-mesons and electrons was small and did not require great accuracy in its measurement; the correction for multiple scattering for the lowest point of the spectrum was ~ 1.15, and for the remaining points lay within $1.05 \div 1.0$.

The π^+-meson spectrum in the c.m.s. is presented in Fig. 1; the measurement errors are also shown there, taking into account all sources of uncertainty: statistics, the introduction of corrections, the measurement of the μ and e admixture, calibration measurements, and instability of the apparatus.

The liquid-hydrogen target ([8]) used in the experiments had the form of a cylinder 20 mm in diameter. In this case the resolution in the half-height of the peak from reaction (b), taking all errors into account, was $\Delta p/p = \pm 0.027$. Calculation of

Fig. 1. Momentum spectrum of π^+-mesons from reactions (a) and (b) in the c.m.s. of two protons. Points are experimental data; the solid curve is the calculated spectrum of π^+-mesons ([8]); the hatched region is the error corridor.

Figure 1: Fig. 1. Momentum spectrum of π^+-mesons from reactions (a) and (b) in the c.m.s. of two protons. Points are experimental data; the solid curve is the calculated spectrum of π^+-mesons ([8]); the hatched region is the error corridor.

the resolution curve showed that only reaction (a) contributes to the spectrum of π^+-mesons with $p^* < 0.92\, p_0^* = 230$ MeV/c, where $p_0^* = 250.5$ MeV/c is the mean momentum of the π-mesons from reaction (b) in the c.m.s.

Fig. 1. Momentum spectrum of π^+-mesons from reactions (a) and (b) in the c.m.s. of two protons. Points are experimental data; the solid curve is the calculated spectrum of π^+-mesons ([8]); the hatched region is the error corridor.

By subtracting the differential cross section of reaction (b) at an angle 53° ([6]), the cross section of reaction (a) for $\theta = 54°$ in the c.m.s. was found to be

$$\frac{d\sigma}{d\omega}(\pi^+ pn) = (0.88 \pm 0.09) \cdot 10^{-27}\ \mathrm{cm}^2/\mathrm{sr},$$

and, correspondingly, the total cross section of reaction (a) is

$$\sigma(\pi^+ pn) = 4\pi\, \frac{d\sigma}{d\omega}(\theta = 54^\circ) = (11.1 \pm 1.1) \cdot 10^{-27}\ \mathrm{cm}^2,$$

which is in good agreement with the data of other authors ([9,10]).

The spectrum of π^+-mesons at an angle of $\sim 54°$ in the c.m.s. and the differential cross section of reaction (a) are determined by the expressions

$$\frac{d^2\sigma}{d\omega\, dQ} = \sum_{l=1}^{3} \sum_{J=0}^{3} |\beta_{Jl}|^2\, a_{Jl}^+(Q); \tag{8}$$

$$\frac{d\sigma}{d\omega} = \sum_{l=1}^{3} \sum_{J=0}^{3} |\beta_{Jl}|^2 A_{Jl}^+, \tag{9}$$

since $P_2(\cos\theta) = 0$ for $\theta \approx 54°$. The cross section of reaction (b) is determined by only one parameter:

$$\sigma(\pi^+ d) = 4\pi |\beta_{22}|^2 A_{22}^d. \tag{10}$$

The complete set of measurement results included 12 values of the π^+-meson spectrum from reaction (a) and the values of the total cross sections $\sigma(\pi^+ pn)/4\pi$ and $\sigma(\pi^+ d) = (3.0 \pm 0.23) \cdot 10^{-27}$ cm^2 ([5,10]).

The search for the best version of the set of independently varied parameters $|\beta_{Jl}|^2$ was carried out with the aid of a high-speed computer of the Joint Institute for Nuclear Research. A standard computation program using the "gradient descent" method ([11]) was used to find the minimum of the sum of weighted squared deviations

$$\chi^2 = \sum_{j=1}^{14} \left(\frac{\varepsilon_j}{\Delta_j} \right)^2, \tag{11}$$

where ε_j is the deviation of the calculated j-th value of the quantity under consideration from the corresponding experimental value, determined with error Δ_j.

The best agreement was given by the version of the set in which the parameters of the P-wave are connected by the relations (7), where

$$|\eta_{01}|^2 = |\eta_{11}|^2 = |\eta_{21}|^2 = |\eta_{23}|^2 = |\eta|^2, \tag{12}$$

and the values of δ_{Jl}^R were taken from work ([2]). As a result the following were found: $|\eta|^2 = 0.199 \pm 0.018$; $|\beta_{22}|^2 = 0.013 \pm 0.001$; $|\beta_{33}|^2 = 0.060 \pm 0.012$ with $\chi^2/\overline{\chi}^2 = 1.25$. This corresponded to the total meson-production cross section

$$\sigma_t = (18.6 \pm 0.5) \cdot 10^{-27} \text{ cm}^2$$

and to the production cross section of π^0-mesons from reaction (c)

$$\sigma(\pi^0 pp) = (3.27 \pm 0.25) \cdot 10^{-27} \text{ cm}^2.$$

For an independent separation of the P-wave parameters (except for the $^3F_3$ state), the measurements of the spectra and cross sections for π^+-meson production are insufficient.

From the values found for the parameters $|\beta_{Jl}|^2$, the absorption coefficients in pp scattering were calculated:

$$r_{01} = 0.89 \pm 0.06; \qquad r_{11} = 0.55 \pm 0.10; \qquad r_{21} = 0.77 \pm 0.03;$$

$$r_{22} = 0.72 \pm 0.03; \qquad r_{23} = 0.89 \pm 0.01; \qquad r_{33} = 0.88 \pm 0.03.$$

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Note: Figure translations are in progress. See original paper for figures.