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Abstract

Full Text

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MATHEMATICS

R. A. KORDZADZE

ON SINGULAR INTEGRAL EQUATIONS WITH SHIFT

(Presented by Academician I. N. Vekua, 7 VII 1964)

Let Γ be a collection of a finite number of simple closed Lyapunov contours having no common points; let $\alpha(t)$ be a function given on Γ which maps Γ homeomorphically onto itself, preserving the direction of traversal, and has a derivative $\alpha'(t) \in H$, $\alpha'(t) \neq 0$ ($t \in \Gamma$). In addition, it is assumed that $\alpha(t)$ generates a cyclic group of order n , whose elements we shall denote by

$$\alpha_0(t) \equiv t, \quad \alpha_1(t) \equiv \alpha(t), \dots, \alpha_{n-1}(t).$$

In the paper ⁽²⁾ the singular equation with shift

$$T\varphi \equiv \sum_{l=0}^{n-1} \left\{ A_l(t_0)\varphi(\alpha_l(t_0)) + \frac{1}{\pi i} \int_{\Gamma} \frac{K_l(t_0, t)\varphi(t) dt}{t - \alpha_l(t_0)} \right\} = f(t_0)$$

was considered (we use the notation and numbering adopted in ⁽²⁾), as well as the adjoint equation $T'\psi = 0$.

In the proof of Theorem 6 an inaccuracy was made, to which N. E. Tövmassian drew our attention. As a consequence, the index formula given in ⁽²⁾ requires clarification. The remaining results of the paper remain valid. Below we give the proof of Theorem 4 and the index formula for the operator T .

We shall assume that the operator T is of normal type; hence, as is easy to see, the operator T' is normal.

Let us prove Theorem 4. As shown in ⁽²⁾, with the equation $T\varphi = f$ ($T'\psi = 0$) there is associated a system of singular integral equations

$$\vec{D}\vec{\Phi} = \mathbf{F} \quad (D^*\mathbf{X} = 0),$$

where

$$\mathbf{F}(t_0) = \{f(t_0), f(\alpha(t_0)), \dots, f(\alpha_{n-1}(t_0))\}. \quad (1)$$

To prove Theorem 4, by virtue of Theorem 5, we need to show that from the conditions (3.2) there follows the solvability of the system $\vec{D}\vec{\Phi} = \mathbf{F}$, and conversely (see ^{1, 2}). Let $\psi^\delta(t)$ ($\delta = 1, 2, \dots, k'$) be a complete system of linearly independent solutions of the equation $T'\psi = 0$. Then the vectors

$$\{\psi^\delta(t), \psi^\delta(\alpha(t)), \dots, \psi^\delta(\alpha_{n-1}(t))\}$$

will be solutions of the associated system of equations $D^*\mathbf{X} = 0$, and the vectors

$$\mathbf{X}^\delta(t) = \{\psi^\delta(t), \alpha'(t)\psi^\delta(\alpha(t)), \dots, \alpha'_{n-1}(t)\psi^\delta(\alpha_{n-1}(t))\} \quad (\delta = 1, 2, \dots, k')$$

will be solutions of the system $D'\psi = 0$, adjoint to the system $\vec{D}\vec{\Phi} = 0$. Taking into account the equalities

$$\int_{\Gamma} \mathbf{F}(t)\mathbf{X}^\delta(t) dt = n \int_{\Gamma} f(t)\psi^\delta(t) dt \quad (\delta = 1, 2, \dots, k'),$$

we immediately obtain the conditions (3.2) as necessary.

Let us prove the sufficiency of the conditions (3.2). Let

$$\mathbf{y}^\delta(t) = \{y_0^\delta(t), \dots, \dots, y_{n-1}^\delta(t)\} \quad (\delta = 1, 2, \dots, \tilde{k}')$$

be a complete system of linearly independent

solutions of $D'\vec{\psi} = 0$. Then, as is not difficult to verify, the vectors

$$\left\{ y_0^\delta(t), \frac{1}{\alpha'(t)} y_1^\delta(t), \dots, \frac{1}{\alpha'_{n-1}(t)} y_{n-1}^\delta(t) \right\}$$

will be solutions of the system $D^*X = 0$, and the functions

$$\omega^\delta(t) = y_0^\delta(t) + \frac{1}{\alpha'_{n-1}(\alpha(t))} y_{n-1}^\delta(\alpha(t)) + \dots + \frac{1}{\alpha'(\alpha_{n-1}(t))} y_1^\delta(\alpha_{n-1}(t))$$

$$(\delta = 1, 2, \dots, \tilde{k}')$$

will be solutions of $T'\psi = 0$. Taking into account the equalities

$$\int_{\Gamma} F(t)y^{\delta}(t) dt = \int_{\Gamma} f(t)\omega^{\delta}(t) dt \quad (\delta = 1, 2, \dots, \tilde{k}'),$$

by virtue of (3.2), we obtain that the system $D\Phi = F$ is solvable. Theorem 4 is proved.

Let H be the space of all vectors with n components satisfying the Hölder condition, and let \tilde{H} be the subspace of the space H whose elements are all vectors of the form (1).

Denote by $k, k', \tilde{k}, \tilde{k}'$ the numbers of zeros of the operators T, T', D , and D' , respectively. It is clear that the numbers of zeros of the operators D and $(D^*)'^*$ coincide (see (2)).

Let

$$\vec{\Phi}^{\delta}(t) = \{\Phi_0^{\delta}(t), \dots, \Phi_{n-1}^{\delta}(t)\} \quad (\delta = 1, 2, \dots, \tilde{k})$$

and

$$X^l(t) = \{X_0^l(t), \dots, X_{n-1}^l(t)\} \quad (l = 1, 2, 3, \dots, \tilde{k}')$$

be complete systems of linearly independent solutions of the systems of equations $(D^*)'\vec{\Phi} = 0$ and $D'X = 0$, respectively. Denote by d (d') the maximal number of vectors (if such exist) among $\{\vec{\Phi}^{\delta}(t)\}$ ($\{X^l(t)\}$) orthogonal to the subspace \tilde{H} . Without loss of generality one may assume that these are $\vec{\Phi}^i(t)$ ($i = 1, 2, \dots, d$) ($X^j(t)$ ($j = 1, 2, \dots, d'$)). The components of these vectors satisfy the relations

$$\Phi_0^i(t) + \alpha'_{n-1}(t)\Phi_1^i(\alpha_{n-1}(t)) + \dots + \alpha'(t)\Phi_{n-1}^i(\alpha(t)) \equiv 0 \quad (i = 1, 2, \dots, d),$$

$$X_0^j(t) + \alpha'_{n-1}(t)X_1^j(\alpha_{n-1}(t)) + \dots + \alpha'(t)X_{n-1}^j(\alpha(t)) \equiv 0 \quad (j = 1, 2, \dots, d').$$

From Theorem 4, as is easily seen, (2) implies that $\tilde{k}' = k' + d'$, $\tilde{k} = k + d$, and, consequently, for the index of the operator T we obtain the formula

$$\text{ind } T = \frac{1}{2\pi} \left\{ \arg \frac{\det[G(t) - K(t, t)]}{\det[G(t) + K(t, t)]} \right\}_{\Gamma} - \chi(T), \quad (2)$$

where $\chi(T) = d - d'$.

Applying the results of F. V. Atkinson ⁴, from (2) we obtain:

Theorem. If V is an arbitrary completely continuous operator mapping the Hölder space into itself, then $\text{ind}(T + V) = \text{ind } T$.

In accordance with formula (2), the results of ³ must be modified: x^* should everywhere be replaced by $x^* - \frac{1}{2}\chi(T)$, where $\chi(T)$ corresponds to equation (1.7). In addition, in the conditions of Theorems 1-3 the necessity requirements should be omitted.

Novosibirsk State University

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CITED LITERATURE

- ¹ N. I. Muskhelishvili, *Singular Integral Equations*, Moscow, 1962.
- ² R. A. Kordzadze, DAN, 154, No. 6 (1964).
- ³ R. A. Kordzadze, DAN, 155, No. 4 (1964).
- ⁴ F. V. Atkinson, Mat. sbornik, 28, No. 1 (1951).

* $(D)^*$ is the operator adjoint to the operator D .

Note: Figure translations are in progress. See original paper for figures.

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