A. B. MIKHAILOVSKII, E. A. PASHITSKII

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Abstract

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A. B. MIKHAILOVSKII, E. A. PASHITSKII

HIGH-FREQUENCY DRIFT INSTABILITY OF PLASMA

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Physics

1. In investigations of the stability of an inhomogeneous plasma, it is generally assumed that the frequencies of the unstable oscillations are less than, or of the order of, the ion cyclotron frequency ω_{Hi} or the Langmuir frequency ω_{pi} ([1,2]). For such oscillations the motion of the ions plays an important role, and the problem involves the ratio of the Larmor radius of the ions ρ_i to the characteristic scale of the plasma inhomogeneity a. In this case it is practically possible to study oscillations only in the limit of a small ratio ρ_i/a. (Some steps toward investigating plasma oscillations with arbitrary ρ_i/a were made in work ([3]).)

However, the problem of investigating oscillations in a plasma with non-small ρ_i/a becomes readily feasible if the oscillations considered have frequencies that considerably exceed the ion Larmor and Langmuir frequencies, since then the motion of the ions is insignificant and the oscillations prove to be purely electronic. At the same time such high-frequency electronic oscillations may be of interest for the theory of the stability of a strongly inhomogeneous plasma, when the equilibrium electronic plasma parameters vary over distances much smaller than the ion Larmor radius, $\rho_i/a \gg 1$, since in this case the characteristic drift frequencies $\omega_{\rm dr} \simeq k_\perp T_e/m_e\omega_{He}a$ (T_e and m_e are the temperature and mass of the electrons; ω_{He} is the electron Larmor frequency; $k_\perp$ is the component of the wave vector perpendicular to the magnetic field) are very large, so that the typical condition for drift instability $\omega < \omega_{\rm dr}$ can be satisfied. The present work is devoted to the investigation of such high-frequency oscillations in a strongly inhomogeneous plasma.

It must be noted that, owing to the approximate quasineutrality of the equilibrium state of the plasma, we cannot regard the characteristic scale of variation of the electron density n_0 as small compared with the ion Larmor radius. Therefore, in speaking of a strongly inhomogeneous plasma ($\rho_i/a \gg 1$), we shall mean that the scale a characterizes the gradient of the electron temperature or, more precisely, the gradient of their mean thermal energy. (A plasma with a large gradient of the electron temperature is typical for experiments on the interaction of electron beams with a plasma ([4]), when in the region through which the beam passes the hottest electrons are concentrated, while at the periphery there is a relatively cold plasma.)

2. Electronic oscillations of an inhomogeneous plasma in a strong magnetic field are described by equation (5)

$$\operatorname{div}(\nabla_\perp \Psi) - k_z^2 \Psi \left[1 + \frac{4\pi e^2}{m k_z} \int \frac{\partial f/\partial v_z}{\omega - k_z v_z}\, dv_z\right] + i\frac{4\pi e^2}{m\omega_H} \int \frac{[\nabla f, \nabla \Psi]_z}{\omega - k_z v_z}\, dv_z = 0. \tag{1}$$

Here Ψ is the potential of the electric field of the oscillations $\mathbf{E}$ (the oscillations are taken to be approximately potential, $\operatorname{rot}\mathbf{E} \approx 0$); $f(v_z)$ is the distribution function—

distribution of electrons over longitudinal velocities v_z; $\omega_H = eH_0/mc$ is the electron Larmor (cyclotron) frequency; e and m are the charge and mass of the electron; c is the speed of light; H_0 is the magnetic-field strength; k_z is the longitudinal component of the wave vector $\mathbf{k}$ (along $\mathbf{H}_0 \uparrow\uparrow Oz$). The magnetic field is assumed to be sufficiently strong, so that $\omega_H \gg \omega_p = (4\pi e^2 n_0/m)^{1/2}$, the electron Langmuir frequency.

To study small-scale oscillations with wavelength much smaller than the characteristic size of the plasma inhomogeneity, $ka \gg 1$, we use the geometrical-optics approximation, setting $\Psi \sim \exp\{ik_y y + i\int k_x(x)\,dx\}$ (the inhomogeneity is directed along the Ox axis). Then the problem reduces to the investigation of an equation relating the frequency and the coordinate-dependent wave vector $\mathbf{k}(x)$:

$$k^2(x) + \frac{4\pi e^2}{m} \int \left[k_z \frac{\partial f}{\partial v_z} + \frac{k_y}{\omega_H}\frac{\partial f}{\partial x}\right] \frac{dv_z}{\omega - k_{zv} z} = 0. \tag{2}$$

3. Consider an inhomogeneously heated plasma with a Maxwellian distribution function of electrons over velocities:

$$f(v_z, x) = n_0[m/2\pi T(x)]^{1/2} \exp\{-mv_z^2/2T(x)\}, \tag{3}$$

where the electron temperature T is a function of the coordinate x, while the density is constant, $n_0 = \text{const}$. In this case equation (2) takes the form

$$k^2 + \frac{1}{d^2}\left[1 + i\sqrt{\pi}\frac{\omega}{k_{zv}T} W\left(\frac{\omega}{k_{zv}T}\right)\right] - i\sqrt{\pi}\frac{\omega_p^2}{\omega_H}\frac{k_y}{k_z}\frac{\partial}{\partial x}\left[\frac{1}{v_T} W\left(\frac{\omega}{k_{zv}T}\right)\right] = 0, \tag{4}$$

$$d(x) = [T(x)/4\pi e^2 n_0]^{1/2}$$

is the Debye radius;

$$v_T(x) = [2T(x)/m]^{1/2}$$

is the mean thermal velocity of the electrons;

$$W(z) = e^{-z^2}\left[1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2}\,dt\right]$$

is Kramp' s function (1).

Under the condition $\omega \gg k_{zv}T$ (hydrodynamic oscillations), from (4) we obtain the dispersion equation of the oscillations

$$\omega^3 - \omega\omega_p^2\frac{k_z^2}{k^2} - k_{yv_{\mathrm{dr}}}\omega_p^2\frac{k_z^2}{k^2} = 0, \tag{5}$$

where

$$v_{\mathrm{dr}} = \frac{c}{eH_0}\frac{dT}{dx}$$

is the electron drift velocity.

We note that equation (5) is analogous to the dispersion equation of low-frequency oscillations, $\omega \ll \omega_{Hi}$, in an inhomogeneously heated plasma, which was studied in Ref. (6).

One of the roots of the cubic equation (5) corresponds to an instability under the condition

$$k_z < \frac{3\sqrt{3}}{2}\frac{k^2 v_{\mathrm{dr}}}{\omega_p} \simeq \frac{3\sqrt{3}}{2}\frac{\omega_p}{\omega_H}\frac{k_y^2 d^2}{a} \qquad (k_z \ll k_y). \tag{6}$$

The growth rate of the hydrodynamic instability of the oscillations is equal to

$$\gamma \equiv \operatorname{Im}\omega \simeq \omega_p\frac{k_z}{k} \simeq k_{yv_{\mathrm{dr}}} \qquad (\gamma \sim \operatorname{Re}\omega). \tag{7}$$

In this case the condition $\omega \gg k_{zv}T$ means

$$k_y d \ll 1; \qquad k_z a \ll \omega_p/\omega_H. \tag{8}$$

The maximum growth rate is attained at the limit of applicability of equation (5), when $\omega \sim k_{zv}T$, i.e., for $k_y d \sim 1$ and $k_z a \sim \omega_p/\omega_H$, and is, in order of magnitude,

$$\gamma_{\max} \simeq \frac{\omega_p}{\omega_H}\frac{v_T}{a}. \tag{9}$$

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

For $\omega < k_z v_T$ the hydrodynamic instability becomes kinetic, and the increment decreases.

Figure 1 gives the stability boundary of the oscillations. We note that in the region $k_y d < 1$, along with the hydrodynamic instability, for $k_z a < \omega_p/\omega_H\sqrt{2}$ there is also a weak kinetic instability with an exponentially small increment.

Fig. 1. Stability boundary of an inhomogeneously heated electron plasma:
$\eta = k_z a \omega_H \sqrt{2}/\omega_p; \quad \zeta = k_y d.$
The instability region lies below the solid curve. Between the dashed and solid curves is the region of kinetic instability with an exponentially small increment.

Fig. 2. Stability boundary of a mixture of inhomogeneous electron plasmas with different temperatures, $T_1 \gg T_2$;
$\eta = k_z a(\omega_H\sqrt{2})/\omega_{p1}; \quad \zeta = k_y d_1; \quad \alpha = [(n_{02}/n_{01})(T_1/T_2)]^{3/2}.$
The instability region for a given α lies below the corresponding curve. (For example, for $\alpha = 8$ the maximum of the curve lies at $\zeta \simeq 1.4$ and is equal to $\eta_{\max} \simeq 0.52$.)

4. Let us consider a mixture of two Maxwellian electron plasmas with constant but different temperatures T_1 and T_2, and with inhomogeneous densities $n_{01}(x)$ and $n_{02}(x)$, such that the total electron density, compensated by the space charge of the ions (the plasma is quasineutral), is homogeneous in space: $n_0 = n_{01}(x) + n_{02}(x) = \text{const}$, and $dn_{01}/dx = -dn_{02}/dx$ (such a mixture is equivalent to a plasma with a gradient of the electron thermal energy). In this case equation (2) is reduced to the form

$$k^2(x) + \sum_{j=1}^{2} \frac{1}{d_j^2}\left[1 + i\sqrt{\pi}\frac{\omega}{k_z v_{Tj}} W\left(\frac{\omega}{k_z v_{Tj}}\right)\right]\left(1 - \frac{\omega_j^*}{\omega}\right) = 0, \tag{10}$$

where

$$d_j(x) = \left[\frac{T_j}{4\pi e^2 n_{0j}(x)}\right]^{1/2}, \quad v_{Tj} = \left(\frac{2T_j}{m}\right)^{1/2} \quad \text{and} \quad \omega_j^* = \frac{k_y c T_j}{e H_0}\frac{\partial \ln n_{0j}}{\partial x}.$$

Let $T_1 \gg T_2$; then equation (10) for oscillations with $\omega \gg k_z v_{T2}$ takes the form

$$k^2 + \frac{1}{d_1^2}\left[1 + i\sqrt{\pi}\frac{\omega}{k_z v_{T1}} W\left(\frac{\omega}{k_z v_{T1}}\right)\right]\left(1 - \frac{\omega_1^*}{\omega}\right) - \frac{k_z^2 \omega_{p2}^2}{\omega^2} = 0, \tag{10'}$$

where $\omega_{p2} = (4\pi e^2 n_{02}/m)^{1/2}$.

For comparison, let us give the dispersion equation of low-frequency ($\omega \ll \omega_{Hi}$) oscillations in an inhomogeneous plasma with cold ions [7]:

$$k^2\left(1+\frac{\omega_{pi}^2}{\omega_{Hi}^2}\right)+\frac{1}{d_e^2}\left[1+i\sqrt{\pi}\frac{\omega}{k_z v_{Te}}W\left(\frac{\omega}{k_z v_{Te}}\right)\right]\left(1-\frac{k_y v_{\mathrm{dr}}^e}{\omega}\right)-\frac{k_z^2\omega_{pi}^2}{\omega^2}=0. \tag{11}$$

Taking into account the identity of the structure of equations (10′) and (11), and using the analysis carried out earlier [7] for equation (11), we arrive at the conclusion that in the mixture of two electron plasmas under consideration

there is an instability of the drift type. In particular, for oscillations with $\omega > k_z v_{T1}$, just as in the preceding case of an inhomogeneously heated plasma, under the condition

$$k_z a < \frac{3\sqrt{3}}{2}\,\frac{\omega_{p0}}{\omega_H}\,\frac{n_0}{n_{01}}\,k_y^2 d_1^2 \qquad \left(\omega_{p0}=\left(\frac{4\pi e^2 n_0}{m}\right)^{1/2}\right)$$

(the electron Langmuir frequency), a hydrodynamic instability of the oscillations with the maximum increment (9) occurs.

For oscillations with a longitudinal phase velocity lying in the interval $v_{T2} \ll \omega/k_z < v_{T1}$, from equation (10′) we obtain the following expression for the frequency ($\omega^2 \gg \omega_{p2}^2 k_z^2/k^2$):

$$\omega \simeq \frac{\omega_1^*}{k_y^2 d_1^2 - i\sqrt{\pi}\,\omega_1^*/|k_z|v_{T1}}, \tag{12}$$

which is valid in the region $k_y^2 d_1^2 \gg 1$, i.e. $\omega \ll \omega_1^*$.

As can be seen, for $k_y d > 1$ there is a kinetic drift instability of the oscillations. Figure 2 gives the stability boundary of a mixture of electron plasmas with different temperatures.

Thus, the high-frequency instability considered in the present work, caused by the gradient of the electron temperature of the plasma, is analogous to the low-frequency drift instabilities investigated earlier[6,7], and in this connection may be called a **high-frequency drift instability**.

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Institute of Atomic Energy
named after I. V. Kurchatov

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Note: Figure translations are in progress. See original paper for figures.