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Abstract

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Amplification in a Recombining Plasma

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Analysis of the amplification of radiation in a highly ionized plasma, inversely populated with respect to energy levels, is associated with taking into account an entire set of relaxation times. A detailed solution of this problem is far from complete; however, in a qualitative consideration of the processes accompanying the decay of such a plasma, it is convenient to distinguish the following three stages in time: 1) a strongly ionized plasma with "instantaneously" cooled free electrons, when the lower levels are still practically unpopulated; 2) a substantially ionized plasma in which a "stationary flow" from the overpopulated upper levels has been established; 3) a weakly ionized plasma.

In a strongly ionized plasma of medium density ($N_e \sim 10^{11} \div 10^{17}$ cm^{-3}), the relaxation times of electrons over discrete levels substantially exceed the time required for establishing the distribution in the continuous spectrum; therefore such rapid cooling of the free electrons proves possible that the initially small populations of a number of discrete levels do not have time to increase appreciably. Thus a medium is formed that amplifies electromagnetic radiation ([1]); its properties are determined by the particular cooling conditions and by the kinetics of recombination. The relaxation of a plasma whose high degree of ionization greatly exceeds the equilibrium value (at the temperature T_e of the free electrons) has been considered in works ([2–4]): recombination begins with the capture of an electron onto one of the upper excited levels in three-body collisions; then the electron passes to lower levels in collisions of the second kind or spontaneously; it may also pass to higher discrete levels or return to the continuous spectrum in a collision of the first kind. Inelastic collisions sharply reduce the role of states that are metastable for radiative transitions; moreover, one may expect a certain similarity of processes in the decay of a highly ionized homogeneous plasma of different composition. We shall consider an optically thin hydrogen plasma. If, after "instantaneous"cooling of the free electrons, their kinetic temperature is maintained unchanged, the equations for the populations of the energy levels take the form:

$$\frac{dN_n}{dt} = -N_e N_n \left[\sum_{m>n} V(n,m) + \sum_{m<n} R(n,m) \right] - N_e N_n B_e(n) +$$

$$+ N_n \sum_{m<n} A(n,m) + N_e \left[\sum_{m>n} R(m,n) N_m + \sum_{m<n} V(m,n) N_m \right] +$$

$$+N_e^3 B_e'(n) + N_e^2 A_e(n) + \sum_{m>n} A(m,n) N_m, \qquad N = N_e + \sum_n N_n. \qquad (1)$$

Here N is the total number of electrons in 1 cm^3 of plasma, N_e is the density of free electrons, and N_n is the number of electrons in 1 cm^3 on the discrete level with principal quantum number n. The quantities $A(n,m)dt$ and $A_e(n)dt$ are respectively equal to the probabilities of the radiative transition $n \to m$ and of spontaneous recombination to level n during the time dt; similarly $V(n,m)$ and $R(n,m)$

are proportional to the probabilities of radiationless transitions inside the atom; $B_\varepsilon(n)$ and $B_\varepsilon'(n)$ are the probabilities of radiationless ionization and recombination.

For densities $N \sim 10^{13} \div 10^{16}$ cm^{-3} and temperatures $kT_e \sim 0.1 \div 0.5$ eV, the general relaxation scheme is qualitatively as follows: after capture of an electron onto an upper level, collisional relaxation predominates, the probability of which decreases with decreasing n, while the role of radiative transitions increases; at the lower levels ($n < n^*$) radiative transitions play the principal role. The relaxation rate may be estimated by the rate of flow through the level n^*; for $n > n^*$ the electron distribution rapidly becomes quasi-equilibrium:

$$\widetilde{N}_n = n^2 N_e^2 \left(\frac{2\pi\hbar}{mkT_e}\right)^{3/2} \exp\left(\frac{E_n}{kT_e}\right). \qquad (2)$$

The first stage of relaxation is characterized by the time τ_1 required for a stationary flow to be established over the excited discrete levels; the order of magnitude of τ_1 is estimated as the smaller of the time of three-body recombination onto the second level and the sum of the relaxation times between neighboring levels; for the N_e and T_e of interest to us, $\tau_1 \sim 10^{-8} \div 10^{-7}$ s.

The second stage begins a time τ_1 after the sharp cooling of the free electrons and ends when

$$\sum_{n=2}^{n_{\max}} N_n(t) \simeq N_e(t);$$

when calculating the populations $\overline{N}_n$ at stage (2) of the "stationary flow," one must set in (1) $d\overline{N}_n/dt = 0,\ n = 2, 3, \dots, n_{\max}$ (see also ([3,4])). The calculation of $\overline{N}_n$ was carried out at the P. N. Lebedev Physical Institute on an M-20 computer; for the coefficients of system (1), modified Born cross sections for collisional processes ([5]) were used; the value of $n_{\max}$ was determined from the Debye radius, and for $n > 10$ it was assumed that $\overline{N}_n = \widetilde{N}_n$. The cross sections for transitions from states $n > 10$ to lower discrete levels were estimated by asymptotic continuation of the formula for three-body recombination. Table 1* lists the populations $\overline{N}_n$ of levels $n = 2 \div 6$, where $\overline{N}_n$ differs substantially from $\widetilde{N}_n$.

The coefficient of negative absorption of radiation with wavelength $\lambda_{n,m}$ (corresponding to the transition $n \to m$) per 1 cm of photon path is equal to

$$\chi_{m,n} \simeq \frac{\lambda_{n,m}^2}{4\Gamma_{m,n}} A(n,m)(N_n - N_m). \tag{3}$$

Let us estimate, for example, $\chi_{5,2}$ for the 1st and 2nd stages of relaxation. The line width is then determined by collisions with ions ([5]): $\Gamma_{m,n} = 12.5(m^2 - n^2)N_e^{2/3}$. For the 1st stage, assuming $N_5 \simeq \widetilde{N}_5$, $N_2 = 0$ at $kT_e = 0.1$ eV, we find $\chi_{5,2}^{(1)} = 2 \cdot 10^{-28} N_e^{4/3}$; consequently, at $N_e = 10^{15}$ cm^{-3} we have $\chi_{5,2}^{(1)} \simeq 4 \cdot 10^{-2}$ cm^{-1}. For the 2nd stage, according to Table 1, at $kT_e = 0.1$ eV, $N_e = 10^{15}$ cm^{-3}, we find $\chi_{5,2}^{(2)} = 2 \cdot 10^{-3}$ cm^{-1}. Thus, during the entire interval of stationary flow ($\tau_2 \sim 10^{-5}$ s), an inverted population is maintained sufficient to create a quantum generator. During the time $\tau_1 \sim 10^{-8}$ s, corresponding to stage (1), amplification in hydrogen plasma makes it possible to obtain generation already over a length of several centimeters. In an optically thin plasma the quantities $\overline{N}_n$ do not depend on N_1; but in real systems at $N_e \geq 10^{14}$ cm^{-3} it is necessary to take account of reabsorption of radiation, which will lead to an increase of $\overline{N}_2$ and $\overline{N}_3$, their dependence on N_1, and to a decrease–

* Collisional cross sections for upper levels, even in the case of hydrogen, have so far been determined only to within factors of $\sim 1 \div 3$. It is therefore of interest to compare the values of $\overline{N}_n$ calculated by us with those calculated in the recently published papers ([4]), where quasiclassical cross sections were used; the corresponding values of $\overline{N}_n$ proved to coincide in order of magnitude.

$\tau_2, \chi_{n,2}^{(2)}, \chi_{n,3}^{(2)}$. It should be noted that hydrogen is not the best medium for a plasma laser, since, owing to the linear Stark effect, the values of $\Gamma_{m,n}$ are large in it; in addition, the absence of metastability of states in a hydrogen plasma reduces the times τ_1 and τ_2.

In equations (1), trapping of radiation, the influence of undissociated molecules and neutral atoms, and also electron attachment are not taken into account. At stage (1) and, to a certain extent, at stage (2), allowance for all these phenomena would be only of a refining nature, meaningless before the cooling method is specified. At stage (3), however, such effects may become determining. The situation is further complicated by the fact that in a weakly ionized plasma the value of radiationless transitions decreases, which enhances the role of metastable states. In contrast to stages (1) and (2), the state of inverted population in a weakly ionized plasma can easily be maintained stationary; it is realized in a gas laser. Let us dwell on one property of a gas-discharge plasma.

Table 1

	$N_e = 10^{12}$	$N_e = 10^{13}$						
	$kT_e = 0.1$	0.2	0.3	0.4	0.1	0.2	0.3	0.4
N_2	$4.6 \cdot 10^4$	$4.7 \cdot 10^3$	$1.8 \cdot 10^3$	$1.0 \cdot 10^3$	$2.2 \cdot 10^7$	$1.2 \cdot 10^6$	$3.6 \cdot 10^5$	$1.7 \cdot 10^5$
N_3	$1.8 \cdot 10^5$	$1.9 \cdot 10^4$	$7.0 \cdot 10^3$	$3.9 \cdot 10^3$	$9.6 \cdot 10^7$	$8.1 \cdot 10^6$	$2.2 \cdot 10^6$	$9.7 \cdot 10^5$
N_4	$1.0 \cdot 10^6$	$7.9 \cdot 10^4$	$2.5 \cdot 10^4$	$1.3 \cdot 10^4$	$8.8 \cdot 10^8$	$2.6 \cdot 10^7$	$6.6 \cdot 10^6$	$2.8 \cdot 10^6$
N_5	$2.3 \cdot 10^6$	$1.8 \cdot 10^5$	$5.7 \cdot 10^4$	$2.8 \cdot 10^4$	$3.8 \cdot 10^8$	$2.4 \cdot 10^7$	$7.3 \cdot 10^6$	$3.4 \cdot 10^6$
N_6	$2.3 \cdot 10^6$	$2.3 \cdot 10^5$	$8.0 \cdot 10^4$	$4.1 \cdot 10^4$	$2.6 \cdot 10^8$	$2.5 \cdot 10^7$	$8.3 \cdot 10^6$	$4.3 \cdot 10^6$

	$N_e = 10^{14}$	$N_e = 10^{15}$						
	$kT_e = 0.1$	0.2	0.3	0.4	0.1	0.2	0.3	0.4
N_2	$9.6 \cdot 10^9$	$3.6 \cdot 10^8$	$1.2 \cdot 10^8$	$4.9 \cdot 10^7$	$2.0 \cdot 10^{12}$	$2.7 \cdot 10^{11}$	$3.2 \cdot 10^{10}$	$2.3 \cdot 10^{10}$
N_3	$3.2 \cdot 10^{10}$	$6.1 \cdot 10^9$	$1.2 \cdot 10^9$	$3.6 \cdot 10^8$	$1.7 \cdot 10^{12}$	$3.5 \cdot 10^{12}$	$3.9 \cdot 10^{11}$	$6.3 \cdot 10^{10}$
N_4	$4.1 \cdot 10^{11}$	$3.9 \cdot 10^9$	$8.7 \cdot 10^8$	$3.5 \cdot 10^8$	$6.5 \cdot 10^{13}$	$4.4 \cdot 10^{11}$	$9.8 \cdot 10^{10}$	$3.7 \cdot 10^{10}$
N_5	$4.7 \cdot 10^{10}$	$2.6 \cdot 10^9$	$7.7 \cdot 10^8$	$3.6 \cdot 10^8$	$5.2 \cdot 10^{12}$	$2.6 \cdot 10^{11}$	$7.8 \cdot 10^{10}$	$3.6 \cdot 10^{10}$
N_6	$2.7 \cdot 10^{10}$	$2.5 \cdot 10^9$	$8.6 \cdot 10^8$	$4.4 \cdot 10^8$	$2.7 \cdot 10^{12}$	$2.5 \cdot 10^{11}$	$8.6 \cdot 10^{10}$	$4.4 \cdot 10^{10}$

Heating and cooling of free electrons in a gas discharge are combined in time: the heater is the electric field, and the cooler of large heat capacity is both the heavy particles (ions, atoms, and molecules) and the walls. At small degrees of ionization, the frequency ν_e of electron-electron collisions falls, and the distribution $N(E)$ of free electrons differs from the Maxwellian one; this is known for high energies $E > 2kT_e$. It is important to note the deviation of $N(E)$ from a Maxwellian distribution at low energies. As the energy decreases, for $E \ll kT$, collisions of an electron with cold heavy particles become increasingly important; the effectiveness of heating collisions with faster electrons and cooling collisions with ions becomes comparable at $E \sim E_0$

$$\frac{\nu_i(E_0)\,\delta}{\nu_e(E_0,\, E > E_0)} \simeq \frac{\sqrt{\pi}}{2}\,\delta \left(\frac{kT_e}{E_0}\right)^{3/2} \left[-\operatorname{Ei}\left(-\frac{E_0}{kT_e}\right)\right]^{-1} \simeq 1, \tag{4}$$

where ν_i is the frequency of collisions with ions; δ is the relative fraction of energy transferred in these collisions. Substituting into (4) the usual parameters of a gas-laser plasma, $kT_e = 8$ eV, $kT_i = 0.04$ eV, we find, for example, for hydrogen $E_0 \sim 0.6$ eV. That is, for $E_0 < 0.6$ eV, collisions with heavy particles lead to a definite excess of the density of cold electrons over the value corresponding to a Maxwellian distribution at temperature T_e, and consequently to enhancement of recombination into the upper discrete levels. The loss of cold electrons due to

the effect of recombination is compensated by their influx as a result of inelastic collisions. Owing to a certain metastability of the upper levels, the populations on them exceed the quasi-equilibrium values, which may be one of the causes of the inverted population of a gas-discharge plasma. This effect should be enhanced under pulsed modulation of the field, which is qualitatively confirmed by experiment. Finally, let us note the importance of "diffusion cooling" (6), which determines the dependence of the amplification on the distance to the walls and accounts for the dependence of the operation of the entire gas laser on the tube diameter; the role of the walls will be weakened by introducing a magnetic field.

It is necessary to emphasize once more that, in analyzing amplification in a nonequilibrium recombining plasma of various degrees of ionization, one must take into account the kinetics of the entire multilevel system; the treatment carried out so far of the electron distribution over only two or three levels (see, for example, (7)) is not justified here.

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REFERENCES

1. L. I. Gudzenko, L. A. Shelepin, *ZhETF*, **46**, 1445 (1963).

2. N. D' Angelo, *Phys. Rev.*, **121**, 505 (1961); S. Byron, R. C. Stabler, P. I. Bortz, *Phys. Rev. Lett.*, **8**, 376 (1962); E. Hinnov, J. G. Hirschberg, *Phys. Rev.*, **125**, 795 (1962).

3. D. R. Bates, A. E. Kingston, R. W. P. McWhirter, *Proc. Roy. Soc.*, A267, 297 (1962).

4. R. W. P. McWhirter, A. G. Hearn, *Proc. Phys. Soc.*, **82**, 641 (1963); D. R. Bates, A. E. Kingston, *Planetary and Space Sci.*, **11**, 1 (1963).

5. I. I. Sobel' man, *Introduction to the Theory of Atomic Spectra*, Moscow, 1963.

6. M. A. Biondi, *Phys. Rev.*, **93**, 1136 (1954).

7. V. R. Bennet, *UFN*, **81**, 119 (1963).

Note: Figure translations are in progress. See original paper for figures.