Reports of the Academy of Sciences of the USSR

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1965

SovietRxiv

Abstract

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Reports of the Academy of Sciences of the USSR

1965. Volume 162, No. 5

PHYSICS

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ON THE QUANTUM THEORY OF THE DECAY OF UNSTABLE ELEMENTARY PARTICLES

(Presented by Academician V. A. Fock on 22 XII 1964)

1. In this work the quantum theory of the decay[1−5] of elementary unstable particles undergoing cascade decay is refined, for example, according to the scheme $M \to m_1 + m$; $m \to m_2 + m_3$, where $m_1,\ m_2,\ m_3$ are stable, and $M,\ m$ are unstable particles.

The initial fundamental point of the whole work is the natural assumption (in fact accepted by everyone)* that unstable particles are just as **elementary** as stable ones, in the sense that the **properties of unstable elementary particles do not depend on the method of their preparation**, i.e., do not depend on the history of their origin.

2. According to the Fock-Krylov theorem[1], the decay law $L(t)$ of unstable elementary particles is $L(t) = |p(t)|^2$, where

$$p(t) = \int_{E_{\min}}^{\infty} \omega(E) \exp\left\{-\frac{i}{\hbar} E t\right\} dE, \qquad E_{\min} \geq 0, \tag{1}$$

and $\omega(E)$ is the time-conserved (this is precisely the law of conservation of energy in quantum theory) energy (mass) distribution density, which, together with the other quantum numbers, completely characterizes the properties of unstable elementary particles.

Ordinary unstable particles, decaying by means of the weak interaction, live so “long” ($\sim 10^{-10}$ sec) that they can be completely characterized[2] by the pole term in $\omega(E)$**.

$$\omega(E) = C\left[(E - E_0)^2 + \Gamma^2\right]^{-1}, \tag{2}$$

where $\Gamma/E_0 \sim 10^{-15}$. Therefore experiments on resonance scattering of decay products correspond to such small cross sections that, at the present level, carrying them out is practically impossible. Moreover, the accuracy (resolving power) in energy (mass) in modern experiments with elementary particles is extremely low (~ 1 MeV) for detecting effects caused by Γ. In short, in all modern work on the scattering and transformation of elementary particles, unstable particles decaying by means of the weak interaction are regarded as stable, with mass (energy) m_0 (E_0). And only in nonstationary (time) experiments on decay do Γ and, in general, the detailed form of $\omega(E)$ play a fundamental role. For a large number of recently discovered very short-lived resonance particles, the detailed form of $\omega(E)$ has

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* In essence, all experimental methods for studying the properties of unstable elementary particles are based on the assumed validity of this assumption.

** In (2), as usual, it is assumed that $\omega(E)$ has a pole of first order; however, as a detailed investigation shows[(6)], *a priori* this by no means follows, and the possible consequences of the fact that $\omega(E)$ has a pole of higher order[(6)] will be considered below.

is of substantial importance also for stationary experiments on scattering and interconversion of these particles. Unfortunately, time-dependent experiments on their decay are impossible at the present level.

3. Usually, when one considers the decay of an unstable particle $m \to m_2 + m_3$ into stable m_2, m_3, the law of conservation of energy is applied in the traditional form (as also in stationary scattering experiments)

$$E_m = E_{m_2} + E_{m_3}. \tag{3}$$

However, since m is an unstable particle, it does not have a **definite** value of energy (mass), and, consequently, formally (3) is not a law of conservation of energy, but a definition of E_m. What, then, is checked according to (3)? Since ordinary unstable particles live relatively long, i.e. $\Gamma/E_0 \ll 1$, $\omega(E)$ is "almost" $\delta(E - E_0)$, and, consequently, the substantive meaning of (3) consists in the fact that the **sum of the energies of the decay products** $(E_{m_2} + E_{m_3})$ is equal to $E_m^{(0)}$, i.e. to the "mean" value (^{2}) of the energy of the unstable particle m. Strictly speaking, however, the law of conservation of energy is the constancy in time of $\omega(E)$, and not only E_0, and therefore it is of interest to clarify the rigorous consequences of the law of conservation of energy in quantum theory.

4. Let us first consider a single decay $m \to m_2 + m_3$. If E_{m_2}, E_{m_3} are the energies of the stable particles m_2 and m_3, then, according to the law of conservation of energy,

$$\omega_m(E) = \int_{E_{\min}}^{\infty} \omega_{m_2}(E_{m_2})\, \omega_{m_3}(E - E_{m_2}; E_{m_2})\, dE_{m_2}, \tag{4}$$

where $\omega_{m_2}(E)$ is the density of the energy distribution of particle m_2, and $\omega_{m_3}(E; E_{m_2})$ is the conditional density of the distribution of the energy E of particle m_3 for a fixed energy E_{m_2} of particle m_2. The energy-distribution densities $\omega_{m_2}(E)$, $\omega_{m_3}(E; E_{m_2})$, determined by the decay interaction, are, of course, of a purely kinematic character and have nothing in common with the energy distribution of unstable particles. If

$$\omega_m(E) = C_m \left[(E - E_m^{(0)})^2 + \Gamma_m^2 \right]^{-1}$$

is the energy distribution of a resting unstable particle, then the energy distribution of this particle moving with velocity $\mathbf{v}$ is:

$$\omega_m^{(v)}(E) = C_m^{(v)} \left[\left(E - E_m^{(0)} \frac{1}{\sqrt{1 - \mathbf{v}^2/c^2}} \right)^2 + \Gamma_m^2 \left(1 - \frac{\mathbf{v}^2}{c^2} \right) \right]^{-1}. \tag{5}$$

5. Let us now consider the cascade decay $M \to m_1 + m;\ m \to m_2 + m_3$. The consequence of the law of conservation of energy is easy to write down, since the energy E of particle M (which is a random variable) is $E = E_{m_1} + E_{m_2} + E_{m_3}$, and, consequently,

$$\omega_M(E) = \int_{E_{\min}}^{\infty} \omega_{m_1}(E'_{m_1})\, \omega_m(E - E'_{m_1}; E'_{m_1})\, dE'_{m_1} \tag{6}$$

where $\omega_m(E_m; E'_{m_1})$ is the conditional density of the distribution of the energy $E_m = E_{m_2} + E_{m_3}$ of particle m for a fixed energy of particle m_1.

Proceeding from our basic assumption that the properties of unstable elementary particles are independent of the method of their preparation, the dependence of $\omega_m(E; E')$ on E' can have only a kinematic character. To simplify the investigation at first, let us consider the case where in $\omega_m(E; E')$ one may neglect the kinematic dependence. Then $\omega_m(E - E'_{m_1}; E'_{m_1}) = \omega_m(E - E'_{m_1})$, i.e. we arrive at a scheme in which E_{m_1} and $E_m = E_{m_2} + E_{m_3}$ are statistically independent, i.e.

$$\omega_M(E) = \int_{E_{\min}}^{\infty} \omega_{m_1}(E')\, \omega_m(E - E')\, dE'. \tag{7}$$

If now the properties of the particles M and m are specified, i.e., $\omega_M(E)$ and $\omega_m(E)$ are specified, then the question arises: is a cascade decay possible? On the basis of (7) this question reduces to the well-known problem of decomposing probability laws (7). Analytically the question is whether there always exists, i.e. for arbitrary $\omega_M(E)$ and $\omega_m(E)$, a positive normalized solution of the integral equation (7), (6). It is not difficult to show that not always. We shall not present here the most general possible solution of this problem, but shall give an important particular case, namely, when $\omega_M(E) = C_M[(E - E_M^{(0)})^2 + \Gamma_M^2]^{-1}$ and $\omega_m(E) = C_m[(E - E_m^{(0)})^2 + \Gamma_m^2]^{-1}$. Solving equation (7) by the Fourier transform method, we obtain

$$\omega_{m_1}(E) = C_{m_1}[(E - E_{m_1}^{(0)})^2 + \Gamma_{m_1}^2]^{-1},$$

where

$$E_M^{(0)} = E_m^{(0)} + E_{m_1}^{(0)}, \tag{8a}$$

$$\Gamma_M = \Gamma_m + \Gamma_{m_1}. \tag{8b}$$

Conditions (8) are necessary conditions for the possibility of a cascade decay. Condition (8a) is nothing other than the law of conservation of energy in the traditional form of type (3), while (8b) gives a new fundamental restriction

$$\Gamma_M \geq \Gamma_m. \tag{9}$$

If one takes into account the kinematic dependence $\omega_m(E; E')$ on E', then, as can be shown,

$$\Gamma_M \geq \Gamma_m E_m^{(0)}/(E_M^{(0)} - m_1 c^2), \tag{10}$$

i.e., in cascade decays, generally speaking, only the following order is possible—first a rapid decay, and then slower decays.

6. The law of cascade decay (9), (10) obtained above, at first glance paradoxical, which is a direct consequence of the law of conservation of energy and of the fundamental assumption of the independence of the properties of unstable elementary particles from the manner in which they are prepared, is well justified* by experimental data in the field of elementary particles ([8–10]) in those cases where the cascade character of the decay has indeed been experimentally proved.

7. The law of cascade decay is manifestly violated in decays in which the presumably intermediate particle is a π^0-meson. However, owing to the extremely short lifetime of the π^0 ($\sim 10^{-16}$ sec), the cascade character of the decay is not observed directly experimentally (for example, in the decay $\Lambda_0 \to n + \pi^0$, what is actually observed is $\Lambda_0 \to n + 2\gamma$, and only

afterward is the cascade character of the decay $\Lambda_0 \to n + \pi^0;\ \pi^0 \to 2\gamma$ assumed). As a detailed analysis shows, because of the low resolving power in energy (mass) of modern experiments, it is impossible to establish the cascade character of the decay of ordinary unstable particles ($\Gamma \sim 10^{-6}$ eV) by direct measurements of $\omega_m(E; E')$.

8. The law of cascade decay is manifestly violated in cascade decays of unstable excited levels of atoms and nuclei. However, this is not surprising, for unstable excited levels of atoms and nuclei are not actually elementary, in the sense that their properties explicitly depend on how they were prepared. This can be seen directly from the Weisskopf–Wigner theory ([11]) of cascade decay of excited levels of atoms and nuclei. One may propose, in principle, a feasible experiment on the cascade decay of excited levels of atoms and nuclei (with the selection of identically prepared excited levels by fixing the energy of the stable particle preceding–

* In particular, T_{Ξ^0} must be no greater than T_{Λ^0}, as indicated by the data ([12]).

of decay), which can directly prove an explicit dependence of their properties (in particular, their decay law) on the method of their preparation.

9. In the case when elementary particles are described by a pole of the n-th order [6], the basic restriction (10) becomes

$$\Gamma_M \geq \Gamma_m \sqrt{2^{1/n} - 1}\, E_m^{(0)} / (E_M^{(0)} - m_1 c^2), \tag{11}$$

which can substantially (for large n) change the concrete estimates of the prohibitions of cascade decays.

If in the future an experimental violation of the law of cascade decay is indeed found for some particles, this will serve as direct proof of the dependence of the properties of unstable particles on their preparation, which sharply contradicts the meaning usually attached to the concept of an elementary particle.

10. An analogous consideration of the consequences of the strict law of conservation of energy in quantum theory as applied to processes of production of short-lived particles (for example, $\gamma + p \to p + \rho^0,\ \rho^0 \to \pi^+ + \pi^-$), in particular, for studying the behavior of the production cross section of these particles in the vicinity of the "threshold," will be presented in a separate paper.

I express my gratitude to Acad. V. A. Fock for his attention and interesting discussion, to the participants of the seminar of the Department of Theoretical Physics of Leningrad State University for interesting discussions, to Acad. Yu. V. Linnik for his attention and discussion of questions connected with the decomposition of probability laws, and to Prof. M. Goldberger for the opportunity to become acquainted with work [6] before publication.

Note added in proof. Of great interest is the experimental result [13] $\Gamma_\rho \simeq 80$ MeV, if the ρ-meson is produced in the cascade decay $A \to \rho + \Pi$, i.e. $\Gamma_\rho(A \to \rho + \Pi) < \Gamma_A \simeq 90$ MeV, as should be the case according to (9), (10). I express my gratitude to Prof. G. Goldhaber and S. Goldhaber for sending the preprint before publication.

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Received
16 XII 1964

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Note: Figure translations are in progress. See original paper for figures.