



Soviet-era science, translated into English

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1965

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Abstract

Full Text

THEORY OF ELASTICITY

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LOSS OF STABILITY OF SHELLS OF REVOLUTION UNDER TORSION

Let a strictly convex shell of revolution be acted upon by a moment M , produced by uniformly distributed tangential forces along the edge of the shell. When the moment M reaches a certain critical value, the shell loses stability and on its surface there arises a system of dents regularly arranged along a parallel (Fig. 1). The problem consists in determining this critical value of the moment M .

In view of the fact that the load sustained by the shell at the moment of loss of stability is stationary, the critical value of M can be determined as the moment sustained by the shell under noticeable buckling. Then the deformation of the shell can be approximated by an isometric transformation of the initial form, and it is enough to take account of the deformation energy only at the boundary of the buckling regions. Per unit length of the boundary γ it is determined by the formula

$$\bar{U} = \frac{2E\delta^2\alpha h}{\sqrt{12(1-\nu^2)}\rho}. \quad (*)$$

Here α is the angle between the tangent plane of the curve γ and the tangent planes of the surface; ρ is the radius of curvature of γ ; h is the change in the normal deflection of the shell in passing through the boundary γ of the buckling region. The remaining quantities have their usual meaning: E is the modulus of elasticity, ν is Poisson's ratio, and δ is the thickness of the shell.

In view of the fact that in the initial stage of postcritical deformation the shell undergoes substantial changes only at the boundary of the buckling regions, the finite bending of the shell surface outside a small neighborhood of the indicated boundary may be replaced by an infinitesimal one. The bending fields of infinitesimal bendings inside the buckling regions and outside these regions are subject on the boundary to a special matching condition, which follows naturally from the fact that the deformation of the shell is close to an isometric transformation. As a result, for the deformation of the shell in the zone of substantial changes of form one obtains an explicit analytic expression. We shall not give this expression; we note only that the deformation energy, determined per unit

length of the boundary by formula (*), when computed for the whole boundary of all n buckling regions, will be

$$U = \frac{2E\delta^2\pi}{\sqrt{12(1-\nu^2)}\sqrt{R_1R_2}} (\lambda^4 + \mu^4 + 4\lambda^2\mu^2) \sigma n.$$

Here R_1 and R_2 are the principal radii of curvature at the centers of the buckling regions. The parameters λ and μ characterize the form of the buckling regions, and σ the magnitude of the deflection in the regions. The formula has been obtained under the assumption that the buckling regions are small, have an elliptical form, and are arranged sufficiently densely (n large).

Next we determine the work done by the moment M ,

$$A = M\varepsilon,$$

where ε is the angle of twist of the shell. The quantity ε is also expressed in terms of the parameters characterizing the buckling regions, and for it one obtains the formula

$$\varepsilon = \frac{24\pi \sin 2\vartheta \lambda \mu (\lambda^2 + \mu^2) \sigma}{\rho \Delta y}.$$

Here ϑ is the angle at which the axes of the buckling regions are inclined to the meridian; ρ is the radius of the parallel on which the centers of the buckling regions are located; Δy is the distance between the centers of these regions.

From the condition of equilibrium of the shell,

$$\frac{d}{d\sigma}(U - A) = 0,$$

we obtain a relation for the moment M taken up by the shell during buckling. Taking into account that $n\Delta y = 2\pi\rho$, this relation can be written in the form

$$\frac{\pi\rho^2 E\delta^2}{\sqrt{12(1-\nu^2)}\sqrt{R_1R_2}} (1 + 2\omega^2) - 3 \sin 2\vartheta \omega M = 0,$$

where $\omega = \frac{\lambda\mu}{\lambda^2 + \mu^2}$. The critical value of M is the smallest value determined by this relation. It is obtained for $\omega = 1/2$ and $\vartheta = 45^\circ$. Thus, the critical value of M is determined by the formula

$$M_k = \frac{\pi\rho^2 E\delta^2}{\sqrt{12(1-\nu^2)}\sqrt{R_1R_2}}.$$

Loss of stability is accompanied by the formation of strongly elongated dents on the surface of the shell ($\omega = 1/2$), inclined to the meridian at an angle $\vartheta = 45^\circ$ (Fig. 1).

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Received
27 XI 1964

REFERENCES

1. A. V. Pogorelov, DAN, **159**, No. 6 (1964).

Note: Figure translations are in progress. See original paper for figures.

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